AN APPROACH TO A GENERAL VIEW ON TOLERANCES IN MECHANICAL ENGINEERING

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ABSTRACT
In this paper an approach will be made to a general view on tolerances in mechanical engineering, especially on tolerancing and optimising tolerance specifications by five major strategies.

To improve the quality of a product the tolerancing problem may not be seen isolated, also adjoining subjects have to be considered, as the stiffness and elastic deformation of machine parts, the construction structure, manufacturing processes, the assembly of components and the use of a product (multi-criterial optimisation). One of the possibilities is the use of tolerance controlled constraints.

These consequences of the product tolerancing will have to be investigated by the designer during the design process. Therefore, computer supported tools, which are integrated into an Engineering Workbench (overall architecture) to support the activities of the designer in each stage of the product development process, help to solve design conflicts.

KEYWORDS
Engineering Workbench, Tolerancing, Tolerance Analysis, Tolerance Enlargement, Tolerance Decoupling, Tolerance Avoiding, Tolerance Controlled Constraints for FEA

INTRODUCTION
With the aid of an Engineering Workbench one should be able to find the best solution of a design to requirements. This can be done by integrated synthesis and multi-criterial analysis tools in the meaning of predictive engineering. [1,2]. Criteria in this field are tolerancing, calculation of stiffness and the optimisation.

Tolerances have a leading role in engineering, because no manufacturing process can be effected without deviations. Furthermore defined tolerances guarantee the interchangeability of parts. Technical systems are getting more complex in the development of engineering systems. Therefore one can find the tendency to decrease tolerance zones as a consequence, which is the wrong way. The right way must be to increase tolerance specifications by keeping the correct function, which would mean making the design robust due to deviations of process chains.

Coupling tolerances (clearance, deviation) and the calculation of stiffness (elastic deformation, subsuming thermal) of a product will give a correlation of both as a multi-criterial influence. Therefore, the tolerance controlled constraints will give the designer a calculation model closer to reality.

TOLERANCES IN THE PRODUCT DEVELOPMENT PROCESS
When discussing product deviations we have to see the design process, the construction structure, manufacturing processes, the assembly of components and also the use of the product (Figure 1) [3,4,5].
Requirements to a product, specified in the requirements list (planning stage) have great influence to the selected solution in the design process, e.g. smooth and noiseless running of a gear, gap between machine parts, accuracy of toggling of machine parts. And we find several methods for the calculation or simulation of deviation influences in design: Monte-Carlo simulation, calculation of error relations, statistical (convolution integral) and arithmetical tolerance chain calculations for 1D/2D/3D and linear or non-linear problems.

![Diagram of Quality Management](image)

Figure 1: Tolerances in the Product Development Process

Geometric tolerances are one of the main reasons for quality problems in industrial production [5]. Production distributions (e.g. needle, Gaussian, triangle, symmetric, asymmetric) of various types (Figure 1) can be found in industrial manufacturing processes of large lot sizes [6]. These distributions result from the four factors men, method, material and machine in various forms. So for example we have the influence of heat and pressure in production processes, as welding or injection molding, which result in distortion of machined parts.

The accuracy of components has to be tolerated depending on the level of automation (Figure 1) [7]. For automatic assembly tolerance zones have to be smaller than for selective assembly.

Focussing on the use of a product (Figure 1), the smooth and noiseless run, elastic, dynamic, thermal deviations, vibrations, subsidence, efficiency, performance and the wear have its cause in tolerances or have to be seen in accordance to them [5]. Unbalances of high-speed machine parts and the overlap of deviations of complex mobile technical systems (wheel suspension) will lead to oscillations that influences the comfort. For instance the internal bearing clearance will decrease from an uninstalled over an installed bearing to a bearing at operating conditions.

**APPROACH TO A GENERAL VIEW ON TOLERANCES**

Five approaches can be made to increase the quality, economy and robustness of products (The robustness can be defined as the measure for insensibility of a design towards deviations of manufacturing processes.), whereby the development tendencies of technical systems lead to products with a higher degree of efficiency.

These five steps are:

- tolerancing,
- tolerance analysis,
- tolerance enlargement,
- tolerance decoupling and
- tolerance avoiding.

Each point will be discussed in the following.
1 TOLERANCING

Items of a component, which are important for its function, have to be tolerated by type and value to assure the correct function of mating parts. In literature several methods for tolerancing of geometric part surfaces can be found.

![Tolerancing Method](image)

Figure 2: Step One – Tolerancing [8]

An approach for tolerancing [8] is the determination of functional relevant tolerances from requirements. Figure 2 shows the tolerancing of a shaft-hub-combination. The calculation of the maximum/minimum fit related to the requirement of transferring a torque of 20 Nm by friction engagement leads to dimension tolerances of H7/s6 (DIN ISO 286) and to a cylindrical tolerance of 0.05mm (DIN ISO 1101). The cylindrical fit can be seen as a tolerance chain containing four elements (dimension and form tolerance of shaft, dimension and form tolerance of hub). In this case we also see the need of simultaneous tolerancing and calculation of stiffness, because the dimensioning and selection calculation [9] of the shaft as well as the calculation of the fitting tolerances have to be considered.

![Example](image)

**Example:**

- decision to transfer a torque of 20 Nm by cylindrical fit
- automatical generation of design feature

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![Tolerancing Method](image)

Figure 3: Tolerancing Method [10]

Another method is described in [10]: The eight steps of tolerancing (Figure 3). Relevant elements of a component are marked by the designer. The next steps are the definition of tolerance types (positional tolerances, references, form tolerances) and the variation of tolerance zones. To enlarge tolerance zones, the Maximum-Material-Condition (see also one of the next chapters) should be used. Tolerance types and values are entered into the Technical Drawing or CAD-model.

2 TOLERANCE ANALYSIS AND OPTIMISATION

The tolerance analysis and optimisation can be divided into the part internal analysis and the analysis over all parts (Figure 4) [2].
After the tolerance analysis over all parts, each part with modified tolerance zones has to be calculated again in the part internal analysis. Both can be detailed as follows.

**Tolerance Analysis Part Internal**

The part internal tolerance analysis focuses on the achievable accuracy of manufacturing processes, machines, combination of process sequences and the economy of processes. Predications to the accuracy of manufacturing processes can only be made with uncertainty (uncertainty of manufacturing). Simulation and calculation methods can give the designer the possibility to get information to the expected accuracy of processes (e.g. distortion, uncertainty).

Procedures for determination of the achievable accuracy are coupled with the process. Processes are classified in DIN 8580 (Figure 5) into the categories primary shaping, metal forming, cutting, joining, coating and change of material properties. In literature we find several theoretical approaches and methods to estimate, calculate, simulate or to lean on experimental data for obtaining the accuracy of manufacturing results.

The first way is to use known production scattering from Statistical Process Control for each item of a component and for each type of tolerance. These results can be made available for the designer in databases to calculate the capability of processes.

Second way are numeric simulation methods (linear and non-linear problems) in the field of primary shaping (Figure 5a). The result can be the distortion of the component, whereby the tendency and uncertainty of the process have to be considered, too. Integrating these methods into the design process might be done for modelling after [9], the optimisation of calculated results will need a new concept.

The third way is the modelling and calculation of processes from information of “acceptance terms for machines” from standards (DIN 8601 and VDI/DGQ 3441) [12] (Figure 5b).

In the fourth group joining, e.g. welding, we have to consider the default, resulting from residual stress of temperature interference in the welding process and the welding sequence (Figure 5c). Calculating the effect of tolerances in a joined press fit (see next chapter) can give a more detailed view on the interference of deformations from residual stress of the press fit to the behaviour of the part.
Tolerance Controlled Constraints

The designer needs predictions of stiffness and elastic deformation (subsuming thermo-elastic deformation) of components. The deformation results from forces and moments under operating conditions as well as from residual stress of press fits as a superposition of both. Displacements in the contact area (Figure 6b) have their cause in tolerance specifications (Figure 6a) and in deformation of mating components (compatibility condition). The determination of the irregular mating area will be future work. The presented procedure will show the possibility of coupling stochastic irregularities specified by the anticipated value and scattering to the nodes of a FE-mesh (Figure 6c) of a part in the meaning of linear elastic error propagation. The influence of irregularities of manufactured part surfaces to the residual stress of fitting parts has not been taken into the FE calculation method yet.

Figure 6: Tolerance Controlled Constraints

Tolerance controlled constraints (TCC, Figure 6) can give the designer an answer to the question of the influence of deviations to the system behaviour. The modelling of the stiffness calculation model is then closer to reality.

This strategy can be applied to the Finite-Element-Method or Boundary-Element-Method. In this paper the calculation is shown for the Finite-Element-Method.

\[
K \mathbf{u} = \mathbf{F}
\]

From the Finite-Element-Method for linear elastic problems with small deformations the equation is known, where \(K\) is the known stiffness matrix of the system, \(\mathbf{u}\) is the vector of displacements or rotations and \(\mathbf{F}\) is the vector of external forces or moments.

The system of equations can be partitioned in the following way:

\[
\begin{bmatrix}
K_{aa} & K_{ab} \\
K_{ba} & K_{bb}
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_a \\
\mathbf{u}_b
\end{bmatrix}
= \begin{bmatrix}
\mathbf{F}_a \\
\mathbf{F}_b
\end{bmatrix}
\]  

(1)

where the sub-matrices \(K_{aa}, K_{ab}, K_{ba}\), and \(K_{bb}\) are known, the displacements \(\mathbf{u}_a\) are unknown, the displacements \(\mathbf{u}_b\) are known, forces \(\mathbf{F}_a\) are known, forces \(\mathbf{F}_b\) are unknown. The matrices \(K_{aa}\) and \(K_{bb}\) are quadratic. The vector of unknown displacements \(\mathbf{u}_a\) can be extracted from (1) as follows:

\[
\mathbf{u}_a = K_{aa}^{-1} \left[ \mathbf{F}_a - K_{ab} \mathbf{u}_b \right]
\]

with the abbreviation \(x := \mathbf{u}_a\), \(a := K_{aa}^{-1} \mathbf{F}_a\), \(B := K_{aa}^{-1} K_{ab}\) and \(\mathbf{u} := \mathbf{u}_b\) follows:

\[
x = a - B \mathbf{u}
\]  

(2)

The anticipated value for the case of an identical distribution function for each element of the vector \(\mathbf{u}_b\) leads to:

\[
E\{x\} = E\{a - B \mathbf{u}\} = E\{a\} - E\{B \mathbf{u}\} = a - B E\{\mathbf{u}\}
\]
with the abbreviation \( m_x := E\{x\} \) and \( m_u := E\{u\} \) follows:

\[
m_x = a - B m_u \quad (3)
\]

which gives the mean value of the unknown displacements as function of the mean value of constraints.

Calculating (2) – (3) gives:

\[
x - m_x = B m_u - B u
\]

Multiplying this term with the transposed vector \((x - m_x)^T\) and using the operator of anticipated value leads to:

\[
E\{(x - m_x)(x - m_x)^T\} = E\{(B m_u - B u)(x - m_x)^T\}
\]

The left side of this equation is the wanted Covariance-matrix of unknown displacements. Including (2) and (3) to the right side leads to following equation:

\[
Cov = E\{(B m_u - B u)(B m_u - B u)^T\} = E\{[(B m_u - u)] [B (m_u - u)]^T\} = B E\{(m_u - u)(m_u - u)^T\} B^T
\]

With the Covariance-matrix of \( u \) and \( m_u \) we can determine the Covariance-matrix of \( x \) and \( m_x \):

\[
Cov_{(x,m_x)} = B Cov_{(u,m_u)} B^T \quad (4)
\]

For cases with different distribution functions resulting from manufacturing processes we can calculate a superposition of \( M \) constraints, where for each of the \( M \) calculations (\( M-1 \)) constraints are fixed and one node is admitted with a distribution:

\[
u_{a1} = a - B u_{b1} \quad \text{with distribution function } f_1
\]

\[
\vdots
\]

\[
u_{aM} = a - B u_{bM} \quad \text{with distribution function } f_M
\]

Calculating the anticipated value analogous (3) and superposing the anticipated values will lead to:

\[
E\{u_{a1}\} + \ldots + E\{u_{aM}\} = M*a - B \left[ E\{u_{b1}\} + \ldots + E\{u_{bM}\} \right]
\]

The same superposition can be made for the covariance-matrices analogous (4):

\[
\sum_{i=1}^{M} Cov_{a} = B \sum_{i=1}^{M} Cov_{b} B^T
\]

With this method it is possible to calculate the anticipated value and the scattering (in the diagonal of the quadratic covariance-matrix) from stochastic represented contact zones. This will give the designer the possibility to control the influence of mating (machined) items to the whole structure of the part, especially to interesting (functional relevant) items. Subsequently a multi-criterial optimisation of tolerance specifications and stiffness of parts will follow. The influence of error propagation can be weakened by a suitable shape of the part, which will have positive influence to tolerance chains, whereby the flow of force has to be considered, too. This could lead to design guidelines for this multi-criterial aspect.

The next steps in research will be the determination of the covariance-matrix \( u \) and \( m_u \) by tolerance specifications of size, form and position under the tolerance principle as well as the determination of displacements in the contact zone of two components.

**Tolerance Analysis Over All Parts**

The calculation of tolerance chains over all parts will give the designer answers to the question of the degree of influence of each tolerance to the closure tolerance. This analysis should be done in and around each operating point of the product.

First of all one has to know the product structure (Figure 7) with elements of own manufacturing (make) and bought elements and with their dependencies in and between the components resulting from mating parts and the tolerances [14].

Determining the tolerance gap and the quota of each tolerance, tolerance chains will have to be calculated on each important point of the kinematic path of the product for operating conditions (e.g. thermal conditions). This can be done by arithmetic or statistic methods depending on the philosophy. Position and form tolerances as well as the tolerance principle have to be taken into these tolerance chains [15] to get an exact calculation result. Taking the stiffness, resulting from forces, moments or temperatures (kinetic calculation) into account, the summary deviation under real operating conditions can be obtained. This will lead to the sensitivity due to tolerances and stiffness in discrete points of the kinematic path.
After this analysis, the optimisation of tolerance types, tolerance values, the tolerance principle, the shape or a change of the construction structure to integral structure of higher accuracy with other manufacturing processes can follow. The recently calculated tolerances have to be assigned. The next step will be the part internal tolerance analysis for each modified component.

3 TOLERANCE ENLARGEMENT

To enlarge tolerance zones and make them thereby more economic, four possibilities can be stated (Figure 8).

The first way is to calculate the arithmetic and statistic closure tolerance of a tolerance chain, to subsequently use the enlargement factor \( e (e > 1) \), which is the quotient of the arithmetic closure tolerance and the statistic closure tolerance. Each tolerance zone in the tolerance chain can be enlarged by the calculated factor \( e \).

The second way is to use the Maximum-Material-Condition, which couples positional and dimensional tolerances. If the positional tolerance zone is not used completely, the dimensional tolerance zone can be enlarged by the unused zone of the positional tolerance.

The third way is to use special kinds of machine elements, e.g. tolerance husks. These elements allow the designer to increase the tolerance zones of both, shaft and hub.

And the fourth possibility is to determine the static definiteness (degrees of freedom) [13]. A high degree of static indefiniteness leads to a high sensitiveness of tolerances due to function and assembly. A static definiteness would be the best solution, but can not be reached for every kind of design. So the goal is to bring one or more degrees of freedom into the system, as a measure to reduce the accuracy in a system, in order to enlarge the tolerance zones of tolerance specification while keeping the correct function and assembly.
4 TOLERANCE DECOUPLING

Remembering the tolerancing of the shaft and the hub. One very easy chain appears by calculating the mating of the shaft and the hub (two elements for dimensional tolerances). This chain increases if the circuit around several components is calculated to get the closing tolerance. To get a functional correct closing tolerance zone the tolerance zone of each element in the chain has to be decreased, which leads to uneconomic tolerances. The consequence is to separate the tolerance chains.

Figure 9: Step Four – Tolerance Decoupling

A possible way to separate tolerance chains is seen by using physical effects from other departments (e.g. mechanics of fluids, thermodynamics, software components in the meaning of logic control elements) to solve elementarily functions of the function structure (Figure 9). These physical effects can be found in a “Classification Scheme” with the coupled calculation methods for each element.

The hydraulic elements for gap compensation for valves in automobile engines as a self-adapting system [4], for example, achieve a shortening of the tolerance chains. This element decouples the tolerance chain around the camshaft, rocking lever, valve, cylinder head and the bearing of the camshaft. Another example can be the use of elastic deformation, e.g. a spring pile, to bring axial forces in the system for adjusting bearings instead of using distance rings with high runout accuracy.

5 TOLERANCE AVOIDING

Finally the fifth approach is the avoiding of tolerances (Figure 10) in the meaning of trimming. This can be achieved by using new innovative techniques, the Contrary Oriented Innovative Strategy WOIS [16] or the Theory of Inventive Problem Solving TRIZ [17].

Figure 10: Step Five – Tolerance Avoiding

The well known example of the conrod with broken fitting areas will make this clear. The fitting areas had to be grinded to get a good accuracy of fit. With the new technique of breaking the conrod into two pieces, as a technology shift, the accuracy of the fit turned up by “micro fitting” broken surfaces. No tolerances have to be specified for the resulting surfaces, because both broken parts match exactly.
CONCLUSION

In this paper a general view on tolerance problems, as described by the “five points” was presented, which will make tolerance specifications more economic.

And this paper has shown the need for coupling tolerance analysis and the analysis of stiffness calculation to get a multi-criterial potential for product optimisation. The first step to this point is the tolerance controlled constraints for Finite-Element calculations.

The integration of mentioned analysis methods into an Engineering Workbench will give the designer the possibility to select each method in the right stage of design to get the actually needed information.

REFERENCES


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