Manufacturing Tolerance Cost Minimization

Using Discrete Optimization For Alternate Process Selection

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ABSTRACT

The allocation of tolerances among the components of a mechanical assembly can significantly affect the resulting manufacturing costs. Using a cost-vs.-tolerance function for each dimension, the least cost tolerance allocation may be determined analytically by the method of Lagrange multipliers. When alternative manufacturing processes are available for some of the components, the analysis may be repeated for each of the possible combinations of processes to determine the least cost production method as well as tolerances. However, the number of combinations increase geometrically, becoming very large for assemblies of moderate complexity.

Several methods are described for systematically searching for the minimum cost process set without having to exhaustively try every possible combination. Two promising methods have been quantitatively tested and compared to an exhaustive search. The number of combinations and CPU time are greatly reduced.

When a preferred tolerance range is specified for each process, the process search methods have difficulty due to the presence of local minima. A procedure for applying process tolerance constraints was developed which greatly increases the likelihood of finding the absolute minimum cost.

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ACKNOWLEDGEMENTS

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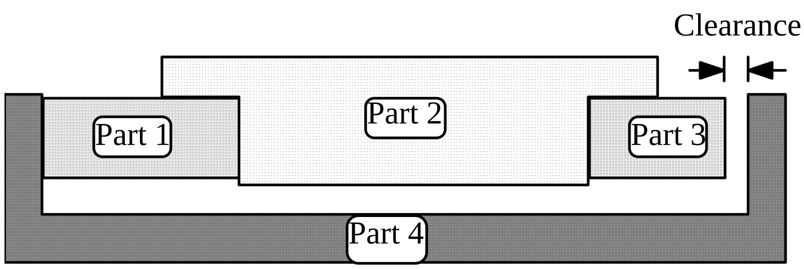
Finally, I thank my wife, Connie, for her support both physical and emotional. She helped keep our personal affairs in order during the final weeks of thesis preparation and very rarely complained. Thanks to my son, James, who still does not know why I always had to leave him. I hope that now we can spend time together every day.

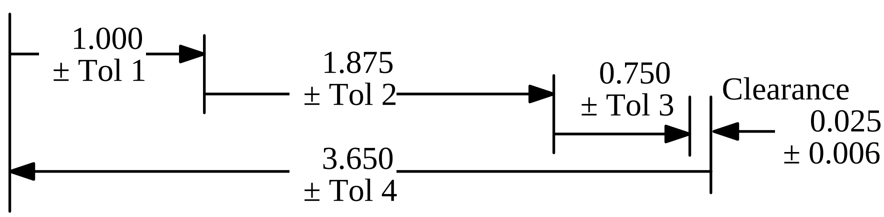
1.0 INTRODUCTION

The specification of tolerances on part dimensions can have a large impact on production cost. By properly allocating tolerances among the components of mechanical assemblies, costs may be lowered without a great deal of overhead. For that reason, tolerance analysis and tolerance allocation for cost minimization is becoming a prominent topic of discussion.

Tolerance analysis is the analytical method of estimating tolerance accumulation in mechanical assemblies. Through tolerance analysis, critical clearances may be maintained and part interchangeability can be assured. For example, given the configuration shown in Figure 1.1 consisting of parts 1, 2, and 3 fitting together and required to fit inside part 4.



a) Sample Assembly



b) Sample Assembly Tolerance Stack

Figure 1.1. Sample Assembly for Tolerance Analysis

Parts 1, 2, 3, and 4 all have dimensions and tolerances associated with them. The tolerance values imply that there is some variability in the actual dimensions of the parts, or from the other side, a tolerance value means that the part dimension can only be assumed to be within a specified range. The sample assembly in Figure 1.1 demonstrates why one must consider the effects of tolerance accumulation to see if the parts will fit together properly.

For tolerance analysis, the assembly tolerance is specified as the overall constraint. In Figure 1.1, the specified assembly tolerance is the tolerance on the clearance. The magnitude of the four componenet tolerances must be assigned by the designer, subject to the assembly tolerance constraint. That is, the sum of the component tolerances, parts 1 to 4, must be less than or equal to the assembly tolerance, clearance, in order to fit.

$$tol_{asm}^{q} \quad \bullet \quad \sum_{i=1}^{n} tol_i^q \qquad \text{(Eq. 1.1)}$$

or

$$tol_{asm} \quad \bullet \quad \left(\sum_{i=1}^{n} tol_i^q \right)^{\frac{1}{q}} \qquad \text{(Eq. 1.2)}$$

N is the number of parts in the assembly. For the case in Figure 1.1, $n = 4$. The q exponent specifies the addition model. When $q = 1$, worst limit tolerancing is used, if $q = 2$, statistical tolerancing results. Statistical analysis assumes a normal distribution of part tolerances, allowing for larger tolerances than with worst limit because of the low probability that all worst case parts will be selected for assembly at once. (Fortini[1])

Given the constraint of Eq. 1.2, all component tolerances could be evenly allocated to spread the available assembly tolerance across all parts. However, each component tolerance may have a different manufacturing cost associated with it due to part complexity or process differences. If tolerance costs are considered in the tolerance allocation, the component tolerances may be allocated to minimize the manufacturing cost of an assembly .

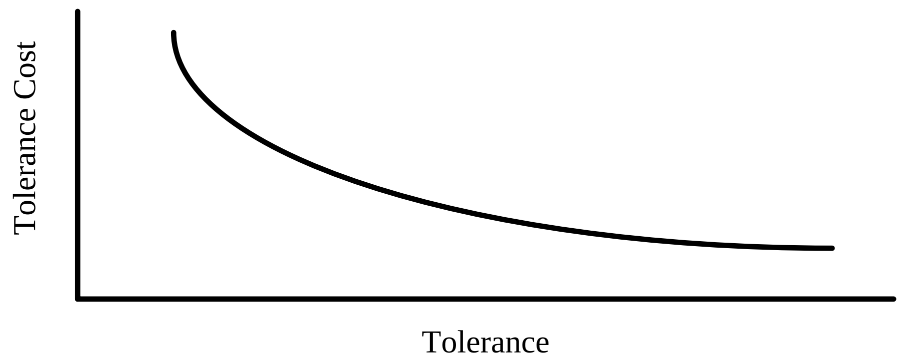


Figure 1.2. Sample Process Cost Curve

The cost of producing a given tolerance is of the form displayed in Figure 1.2. As the tolerance increases, the cost goes down. If more than one process is capable of producing the same part dimension, then the cost curves for each available process for producing the part could be included for comparison. In this way, the least cost manufacturing processes for an assembly may be selected in addition to the minimum cost tolerance allocation.

1.1 Single Process Optimization

Spotts[10] and Speckhart[12] developed models for solutions to minimum cost tolerance allocation. They both assumed that the processes to be used were already selected. Spotts modeled the tolerance cost as varying as the inverse of the tolerance squared, while Speckhart used an exponential form. However, both presented the idea that a closed form solution using Lagrange multipliers was efficient and well suited to the tolerance allocation problem.

The method of Lagrange multiplier optimization of tolerance allocation is also used in the CATS.BYU[2] software. The CATS.BYU program has been under development since 1984 and was first released in 1986. It is a basic tolerance analysis program for linear tolerance stack problems. In the current edition (Version 1.1), cost allocation can be performed given a single process cost curve for each component and requiring the same form for each process cost curve within an assembly.

Because of the great desire to minimize costs in the manufacturing arena, CATS has received a welcome response. However, sponsors of the CATS.BYU program have requested many enhancements; including the ability to handle multiple cost curves to assist in process selection has often been requested and was the impetus behind this master's thesis.

Although the Lagrange multiplier technique is efficient for tolerance allocation, it is important to extend the capability to allow consideration of alternate processes. The Lagrange multiplier technique does not permit alternate process selection by itself. However, it can be combined with other methods to successfully solve the problem.

1.2 Alternate Process Selection Methods

Because the tolerance allocation problem could be solved in closed form, the optimum process selection could be treated as a discrete combinatorial problem. For every combination of process cost curves selected, an assembly tolerance cost could be used as the objective function and compared with other costs from different combinations until the global optimum was discovered. However, for each combination a complete tolerance allocation must be done before the assembly tolerance cost can be compared. Although exhaustively searching (trying every process curve combinations) would provide the global optimum to a problem, it may become too costly or too time-consuming to perform all of the evaluations necessary to check each possibility. Therefore, discrete optimization techniques can be used to lower the number of cases to be checked and thus reduce the computation time and expense.

1.2.1 Balas Algorithm

One method for including process selection in a tolerance allocation problem is the Balas algorithm to solve a completely discrete problem. Huang and Ostwald[11] approached the problem using the Balas algorithm. The Balas algorithm was designed to do a selective search over the entire combination space to find the global optimum. The method involves the following steps.

1) The designer selects combinations of tolerance values and associated process costs to represent the continuous process cost curves at discrete points.

2) An objective function is formed by summing the possible discrete cost terms for all parts in the assembly.

3) Binary coefficients are placed in front of each tolerance-cost value and the coefficients are turned on and off (1 and 0) to select tolerance combinations across the entire problem.

4) Tolerances corresponding to the selected costs are summed and compared to the specified assembly tolerance. Invalid combinations are eliminated.

5) By requiring that all coefficients for one part add up to one, the method is constrained to have only one point for each part activated at a time.

6) Then a systematic search for the minimum cost combination of process-costs satisfying the constraints is done.

The Balas problem may be expressed algebraically as follows:

Minimize the objective function,

$$\text{Cost} = \sum_{i=1}^{n} b_i C_i \qquad \text{(Eq. 1.3)}$$

subject to the constraints,

$$tol_{asm} \bullet \sum_{i=1}^{n} \sum_{j=1}^{m_i} b_{ij} t_{ij} \qquad \text{(Eq. 1.4)}$$

$$\sum_{j=1}^{m_i} b_{ij} = 1 \qquad (i = 1,...,n) \qquad \text{(Eq. 1.5)}$$

$$b_{ij} = \begin{Bmatrix} 1 & \text{on} \\ 0 & \text{off} \end{Bmatrix} \qquad (j = 1,...,m_i), (i = 1,...,n) \qquad \text{(Eq. 1.6)}$$

(n = number of parts, m_i = number of discrete points for $part_i$)

Hauglund[9] compared the Balas algorithm against a continuous approximation method to determine the best method for process selection and tolerance allocation. He also demonstrated a continuous model of the alternate process cost curves. Both methods were compared with the exhaustive search in CPU time necessary for solution as well as accuracy and analysis efficiency. The continuous and Balas methods both proved to be more CPU intensive than the exhaustive search. However, the continuous model did become more efficient than the exhaustive search as the number of process cost curves exceeded thirty-five. A drawback of the Balas algorithm was that discrete points had to be selected from the process cost curves to generate a totally discrete problem. In industry, cost curves tend to be at least piecewise continuous and hence would lose some continuity depending on the number of points used to represent the curves.

Hauglund’s work showed that the exhaustive combinatorial search actually proved more efficient than either the Balas algorithm or the continuous problem representation until the number of curves began exceeding the thirty-five to forty limit. In this case, the number of combinations exceeded one million and the continuous solution became more efficient. In all three methods, the CPU usage was extensive and very costly and viewed as unacceptable for design iterations. Hauglund also had difficulty evaluating problems when constraints were placed on the process tolerance ranges. Process tolerance constraints increased the complexity of the problem and led sometimes to no solution being found. The most difficult case occurs when the tolerance ranges of the alternate process do not overlap, as shown in Figure 1.3. In Figure 1.3, process cost curves for one part are represented as continuous curves; the specified valid range of process tolerances is shaded.

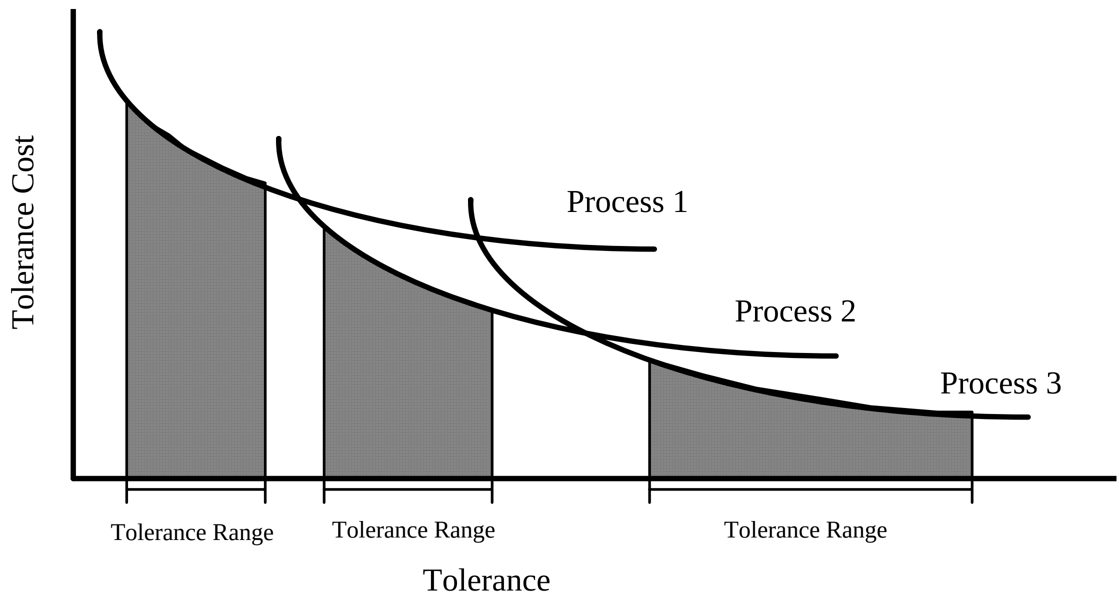


Figure 1.3. Bounded Process Cost Curves

1.2.2 Branch and Bound

Another discrete optimization procedure is the Branch and Bound algorithm. It looked promising as a method for process selection. Branch and Bound algorithms may be used on discrete problems with multiple available discrete combinations if the alternatives can be represented in a tree-like structure. Searches are done across single tree levels. Only those branches that have the best objective function values are continued to be evaluated while the expensive branches are pruned from the search.

Preliminary studies caused us to abandon the Branch and Bound method. Although the multiple processes under consideration for this thesis could be represented by a combination tree, the re-allocation of tolerances for each combination prevented the Branch and Bound method from behaving well. As tolerances were allocated, processes could be switched or out-of-range tolerances would need to be fixed at process limits. This meant that the constraints on the problem could be changing as a function of the tolerance allocation. A better method of considering the effects of tolerance allocation on process selection and overall assembly cost was needed.

1.3 Thesis Objectives

The objectives of this thesis include:

1) Find a more efficient solution method for the combined discrete process selection and tolerance allocation optimization problem arising from alternative process availability.

2) Investigate behavior of methods when non-continuous process tolerance ranges are applied.

3) Recommend a method for use in CATS.BYU.

1.4 Overview of Results

1) Branch and Bound solution method was determined to be unusable for this application.

2) Various new combinatorial search methods were tried. A common set of test problems was used to test each method. Comparisons were made of CPU time and total number of combinations tried to find the minimum cost solution. Three methods tested in detail are called:

   a) Exhaustive search method

   b) Bottom Curve Follower method

   c) Univariate search method

   The exhaustive search was used for comparison.

3) A method for handling process tolerance limits was developed for use with the combinatorial search methods. It is based on first solving the unconstrained problem for a particular process combination, then bringing any out-of-bound tolerances in-bounds by gradually applying the constraints.

4) It was determined that an extended univariate search should be used for problems when the number of exhaustive search combinations exceeds 50. The exhaustive search is still the only method that can guarantee the absolute minimum cost for a given assembly, but the continued univariate search

provided the same accuracy as the exhaustive search on large problems at a fraction of the CPU time.

5) Suggestions for implementation in the CATS.BYU program are included to allow the procedures mentioned in this thesis to be included in the near future.

2.0 MODEL DEVELOPMENT

Development of the ground work for this thesis consisted of deciding on the form of the tolerance cost relationship in order to develop an objective function and then including the tolerance constraints in the problem.

2.1 Assembly Cost Function

The cost verses tolerance equation for a single process was represented in the following form:

$$Cost_i = A_i + B_i \, Tol_i^{\,p} \qquad \text{(Eq. 2.1)}$$

This cost curve model was patterned after the work done by Spotts[10]. The model consists of a fixed cost offset (A) plus a cost coefficient (B) times the tolerance (Tol) raised to a power (p) as shown in equation 2.1. Spotts recommended an exponent of p = -2. Chase and Greenwood[2] recommended p = -1. Chase and Greenwood also noted that they recommended their model on the basis of limited data. However, it is believed that using the same form of the cost equation and allowing non-integer values of p, most process cost curves could be approximated by equations of this form, if only piecewise. Hence, the modified form of the cost function is used exclusively in the development of the theory and problems for this thesis. The generalized cost equation for a given tolerance is then represented in the following form:

$$Cost_i = A_i + B_i \, Tol_i^{\,p_i} \qquad \text{(Eq. 2.2)}$$

The tolerance cost of an assembly is the sum of the individual part costs.

$$Cost_{asm} = \sum_{i=1}^{n} Cost_i = \sum_{i=1}^{n} \left(A_i + B_i \, Tol_i^{\,p_i} \right) \qquad \text{(Eq. 2.3)}$$

(n = number of parts)

Minimizing the assembly tolerance cost entails minimizing equation 2.3 by selection of an appropriate set of component tolerances. Of course, the absolute minimum would occur as the component tolerances approached infinity given no restriction on tolerance magnitudes. However, the component tolerances are constrained to meet some performance criteria.

2.2 Assembly Tolerance Constraint

The performance criteria of a tolerance analysis problem is that the sum of the individual component tolerances raised to the corresponding power must be less than or equal to the specified assembly tolerance raised to that same power.

$$tol_{asm}^{q} \quad \bullet \quad \sum_{i=1}^{n} \left| \frac{f}{x_i} \right|^{q} tol_i^{q} \qquad \text{(Eq. 2.4)}$$

or

$$tol_{asm} \quad \bullet \quad \left(\sum_{i=1}^{n} \left| \frac{f}{x_i} \right|^{q} tol_i^{q} \right)^{\frac{1}{q}} \qquad \text{(Eq. 2.5)}$$

$$q = \begin{Bmatrix} 1: \text{worst case} \\ 2: \text{root sum squared} \end{Bmatrix}$$

Equation 2.5 describes how tolerance accumulates in a mechanical assembly. Common design assumptions are q = 1: worst case analysis, assuming tol_i selected such that they will meet assembly tolerance limits even when all components are produced at their size limits; or q = 2: root-sum-squared analysis, assuming tol_i selected as having random variations in component dimensions, leading to dimensional variations adding statistically as independent variations. The symbol (f) represents assembly function dimension, and the partial derivative with respect to the part dimension (x) in equation 2.5 is the sensitivity of the tolerance stack to each individual tolerance, $\left| \frac{f}{x_i} \right|^{q}$. The sensitivity only arises in 2-D and 3-D problems as the sensitivity of a 1-D problem is equal to 1.0.

2.3 Lagrange Multiplier Optimum Solution

In order to minimize the tolerance cost, the Lagrange multiplier technique can be used on each of the individual part equations.

$$\frac{\partial\,\text{(Obj. function)}}{\partial\,tol_i} + \lambda \frac{\partial\,\text{(Constraint)}}{\partial\,tol_i} = 0 \qquad (i = 1,...,n) \qquad \text{(Eq. 2.6)}$$

Using equations 2.2 and 2.3 from above, equation 2.6 becomes

$$p_i B_i tol_i^{(p_i-1)} + \lambda q \left|\frac{\partial f}{\partial x_i}\right|^q tol_i^{(q-1)} = 0 \qquad (i = 1,...,n) \qquad \text{(Eq. 2.7)}$$

Because λ is the same for each of the equations, it can be solved for in terms of the other variables.

$$\lambda = -\frac{p_i B_i}{q\left|\frac{\partial f}{\partial x_i}\right|^q} tol_i^{(p_i-q)} \qquad (i = 1,...,n) \qquad \text{(Eq. 2.8)}$$

λ may be eliminated by arbitrarily selecting tol_1 as the reference tolerance. Then equation 2.8 becomes

$$-\frac{p_1 B_1 tol_1^{(p_1-q)}}{q\left|\frac{\partial f}{\partial x_1}\right|^q} = -\frac{p_i B_i tol_i^{(p_i-q)}}{q\left|\frac{\partial f}{\partial x_i}\right|^q} \qquad (i = 1,...,n) \qquad \text{(Eq. 2.9)}$$

Each of the subsequent tolerance values may then be specified in terms of tol_1 .

$$tol_i = \left[\frac{p_1}{p_i} \frac{B_1}{B_i} \frac{\left| \frac{f}{x_i} \right|^q}{\left| \frac{f}{x_1} \right|^q} tol_1^{(p_1 - q)} \right]^{\frac{1}{p_i - q}} \quad (i = 2,3,...n) \qquad \text{(Eq. 2.10)}$$

Substitution of the tolerance values into the constraint equation leads to the following equation.

$$tol_{asm}^q = \left| \frac{f}{x_1} \right|^q tol_1^q + \sum_{i=2}^{n} \left| \frac{f}{x_i} \right|^q \left[\frac{p_1}{p_i} \frac{B_1}{B_i} \frac{\left| \frac{f}{x_i} \right|^q}{\left| \frac{f}{x_1} \right|^q} tol_1^{(p_1 - q)} \right]^{\frac{q}{p_i - q}} \qquad \text{(Eq. 2.11)}$$

An iterative solution for the value of tol_1 can then be used to satisfy the constraint equation and determine the set of component tolerances which yield the minimum cost. This method gives a closed form solution to allocating the tolerances and reduces the search to a value of tol_1 that satisfies the tolerance constraint equation. This represents an extension of Spott's work, as he assumed all the $p_i = -2.0$, which did not require an iterative solution for tol_1.

However, the Lagrange multiplier algorithm does not include provisions for process tolerance limits. It also does not work directly with inter-dependent loops, where two or more tolerance analysis loops have some shared parts. Only one assembly can be analyzed at a time. Analyzing each loop separately and then fixing the shared part tolerances at the minimum overall values can be used to overcome this weakness, but Hauglund[9] mentions that it may not be as efficient as a generalized optimization algorithm.

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However, the Lagrange multiplier algorithm does not include provisions for process tolerance limits. It also does not work directly with inter-dependent loops, where two or more tolerance analysis loops have some shared parts. Only one assembly can be analyzed at a time. Analyzing each loop separately and then fixing the shared part tolerances at the minimum overall values can be used to overcome this weakness, but Hauglund[9] mentions that it may not be as efficient as a generalized optimization algorithm.

3.0 SOLUTION METHODS

The model development chapter was concerned with process cost curve definitions and tolerance allocation given a set of processes. The primary focus of this chapter is to extend the optimum tolerance allocation method to include process selection. That is, given a set of alternate processes for each part of an assembly, find the least cost component tolerances and combination of processes.

3.1 Multiple Cost Curve Problem

When multiple cost curves are introduced, the problem becomes much more complex. Multiple cost curves represent alternate processes capable of producing the desired part feature. Each process has a cost curve associated with it as a function of the tolerance achieved. For example, in Figure 3.1 the various processes could be
1 = turning plus grinding, 2 = turning, and 3 = turning plus two finish grindings.

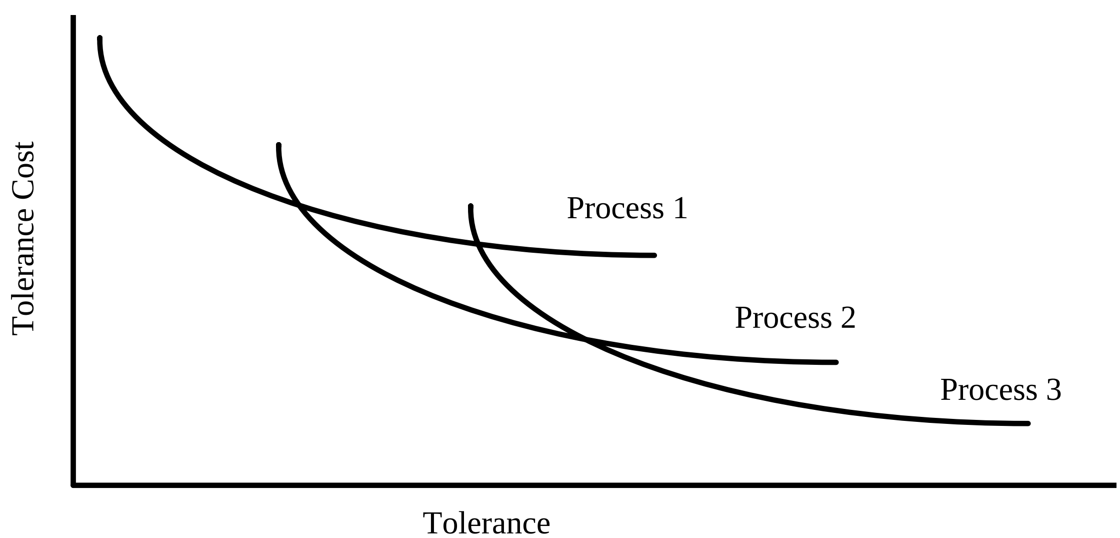


Figure 3.1. Form of Alternate Process Cost Curves

Ideally, all processes for a given feature would have overlapping cost curves that would be continuous over the entire spectrum of tolerances. However, some processes have a limited range of tolerances where they are used. (see Figure 1.3) If an allocated tolerance falls outside the feasible range of a process, another process must be chosen or the tolerance must be fixed at the closest limiting value.

3.2 Experience with Branch and Bound

The OPTDES.BYU program contains a Branch and Bound algorithm to allow discrete variable values to be included for analysis. Initially, a continuous optimum must be found to specify the absolute optimum point available. For the process selection optimization, an iterative search technique is used which varies the exponent p_i and the cost coefficient B_i continuously until it finds an ideal cost curve for each part. The permitted range of p_i and B_i is set such that it includes all of the specified curves for each part. The ideal optimum thus determined is then used as a comparison value from which to start the discrete optimization problem. Various branches of the process tree are evaluated to determine which branches could be eliminated (pruned) from the analysis. The pruning is accomplished by taking one part at a time, substituting an actual process curve, then performing a tolerance allocation and determining the assembly tolerance cost. The selected process is held fixed while you move down that branch of the tree.

When you reach the end of the first branch, all of the ideal cost curves have been replaced by specified process cost curves. The cost at that node is then used as a temporary minimum for comparison with other combinations. Returning to the first part, the next alternative process is selected and then evaluated by re-allocation. If it is not lower in cost than the current minimum cost, the rest of that branch is eliminated from consideration. This procedure is continued until every branch has either been evaluated or eliminated.

The combination of continuous optimum tolerance allocation and discrete process selection did not prove successful. The search surface became so bumpy or "noisy" due to the re-allocation at each node that the search could not proceed to a global optimum value. Many times the algorithm would find local minima and would not be able to go any further. Sometimes it could not even find a feasible solution.

Even though the search method was unsuccessful in OPTDES, contour plots of the assembly tolerance cost as a function of the curve parameters were generated in order to examine the possible gradient patterns exhibited by the process selection problem. A sample plot is shown in Figure 3.2 that shows cost contours for a general tolerance cost function. The circle symbols represent actual processes. Note that the ideal process curve always occurred in the lower right-hand corner. A little thought reveals this to be an obvious result. The iterative search for the ideal curve was thus replaced by a simple scan of the range of p_i and B_i.

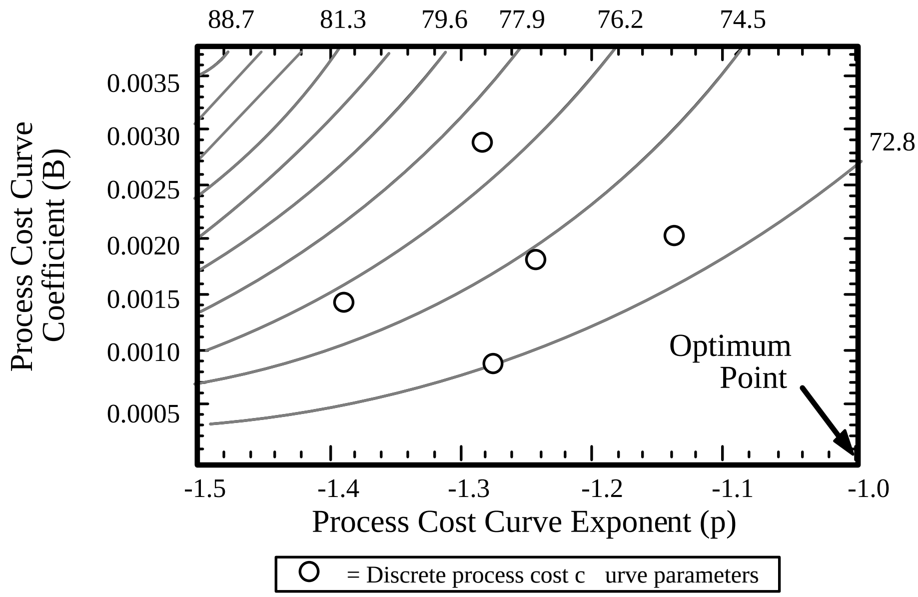


Figure 3.2. Assembly Tolerance Cost Contours

3.3 Exhaustive Search

The exhaustive search evaluates every possible process cost curve combination and a tolerance allocation is computed for each one. It keeps track of the lowest cost combination as it moves systematically through every node in the process tree The number of combinations necessary for evaluation increases geometrically as the number of part processes increased. For an assembly with N parts, each $part_i$ having n_i process curves associated with it, the number of combinations was $n_1 \times n_2 \times n_3 \times . . . \times n_N$. For example, an assembly with three parts that had 2, 2, and 3 curves for parts one, two and three respectively would have $2 \times 2 \times 3 = 12$ combinations to analyze in the exhaustive search. An assembly that had ten parts with each part having just two process cost curves per part would have 2^{10} or 1024 combinations to check. The number of combinations escalates rapidly for large numbers of parts and processes available. One test problem with 13 parts and 38 process cost curves had over a million combinations. Figure 3.4 represents the possible combinations on the three part problem mentioned above and depicted graphically in Figure 3.3 below. The cost figures are included for demonstration purposes only. Note that after a process combination is selected, the tolerances must be re-allocated and possibly have some adjustment of out-of-bound tolerances in order to meet the assembly tolerance constraint. The noisy surface due to tolerance re-allocation did not effect this method since all combinations were evaluated.

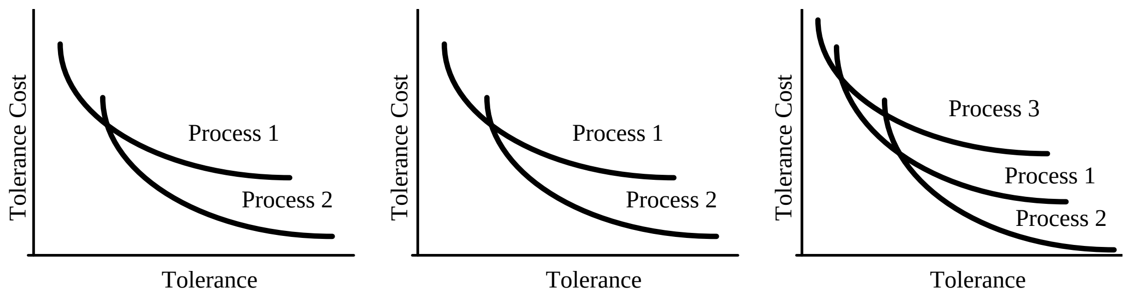


Figure 3.3. Example Problem Process Cost Curves

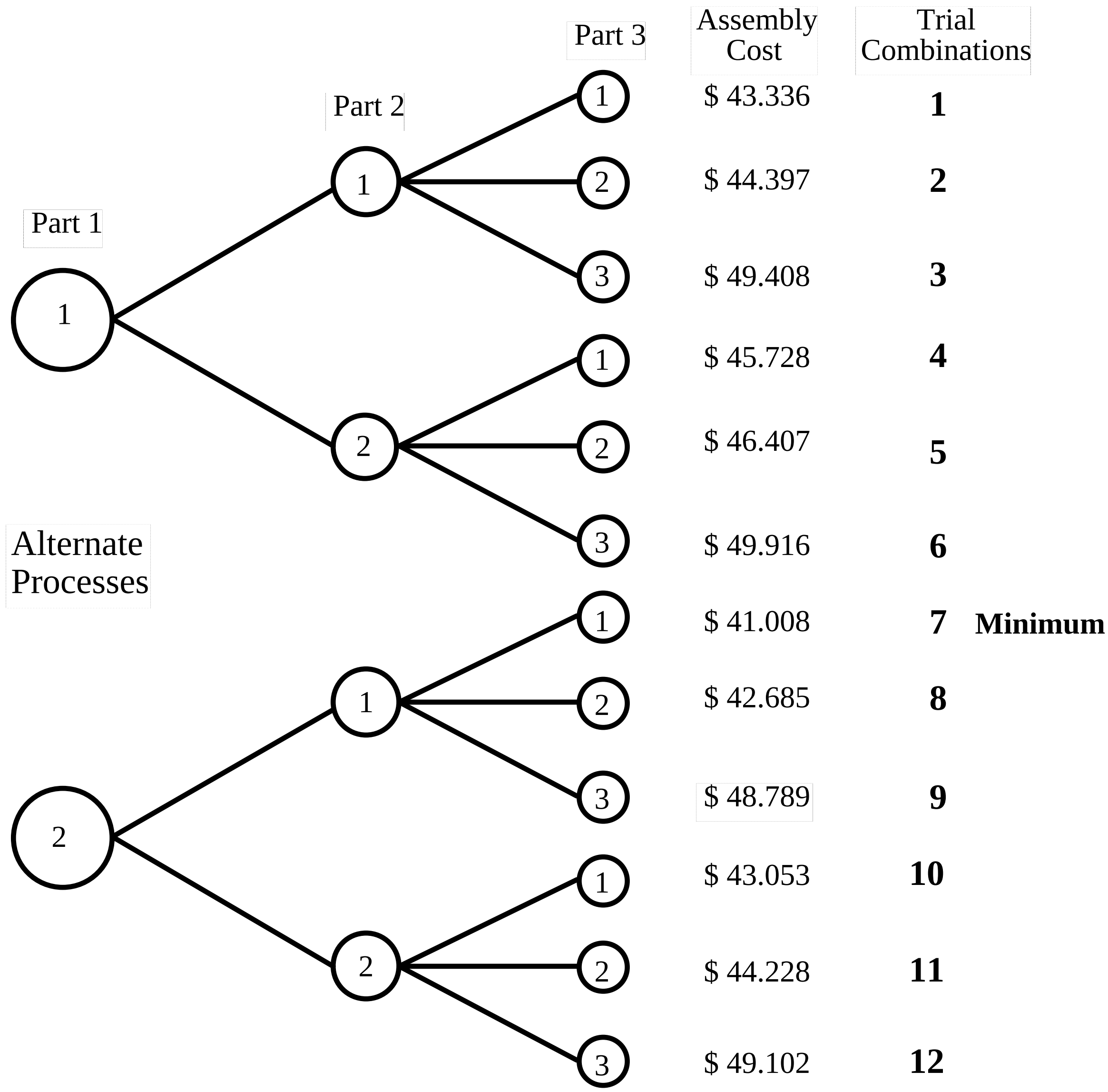


Figure 3.4. Exhaustive Search Tree

3.4 Process Selection by Following the Lowest Curve Profile

Inspection of cost curve plots led to a search procedure in which the lowest cost curves were chosen by a simple evaluation procedure. An initial combination of processes was selected and the tolerances allocated. Then the alternate process cost curves for the first part were compared for a lower cost at the same allocated tolerance value. The lowest process cost curve for the given tolerance was then set as the current process for that part. All parts were evaluated similarly by checking the available processes for each part. If the entire part list was evaluated without any process curve changes, then the process selection was considered complete. Otherwise a new allocation was computed for the revised combination of processes and the evaluation procedure was repeated.

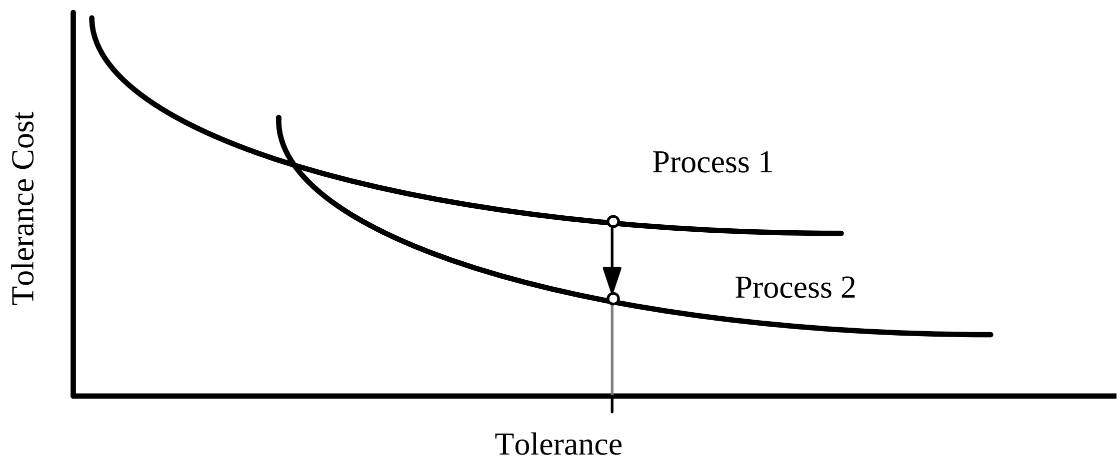


Figure 3.5. Sample Process Cost Curve Switch

In other words, this method would follow the envelope or profile of the lowest process cost curve values as the tolerances were allocated. If no other process cost curve had a lower cost at the current tolerance allocation, the current process was not changed. (see Figure 3.5) By searching the process tree with the tolerances fixed, the noisy surface effects were greatly reduced. This method could always find a near optimum set of tolerances with very few tolerance allocations. However, it did not consider any process tolerance limits. For tolerance analysis cases with wide (essentially unconstrained) process tolerance limits, this could be an efficient solution method.

3.5 Univariate Search with Tolerance Allocation

Through further examination of the exhaustive search trees similar to Figure 3.5 above, distinct assembly tolerance cost patterns were discovered. Because of the patterns that developed in the original test cases, it was decided to use a univariate search to determine the minimum assembly tolerance cost combination of processes. The concept behind the univariate search is that the process tree has cost trends at the end of each tree branch. Because of these trends, a simple search across each of the part levels can reveal the best combination of processes to use. Each part would have its available processes searched once, and the process that produced the lowest assembly tolerance cost was set as the process for that part.

Figure 3.5 demonstrates how a univariate search is performed. First a set of processes are selected, Lagrange multipliers are used to allocate the tolerances, and the assembly tolerance cost is determined. In Figure 3.5, the starting processes are 1, 1, and 1. Then each of the part levels are searched. Part three has each of its alternate processes selected, tolerances allocated, and cost determined. The minimum cost value on the first part level search was with process 1 for part 3. Process 1 is then set for part 3, and the search is continued on part 2. Process 2 for part 2 is selected and the assembly cost is compared with the existing minimum cost. Process 1 for part 2 is set, and the search is moved to part 1. Again, the alternate processes are checked, and process 2 for part 1 is set as the better process. The final solution is processes 2, 1, 1 for parts 1, 2, 3 respectfully.

The univariate search required a minimal number of iterations before finding the solution. The number of process combinations that had to be checked for a univariate search was close to the sum of processes available. Specifically, for an assembly having N parts with n_i process curves for $part_i$, the number of combinations was $n_1+n_2+n_3+...+n_N.- N + 1$. The problem depicted in Figure 3.3, an assembly having three parts with 2,2, and 3 curves for parts one, two and three respectively, would have $2 + 2 + 3 - 3 + 1 = 5$ combinations to check for the univariate search.

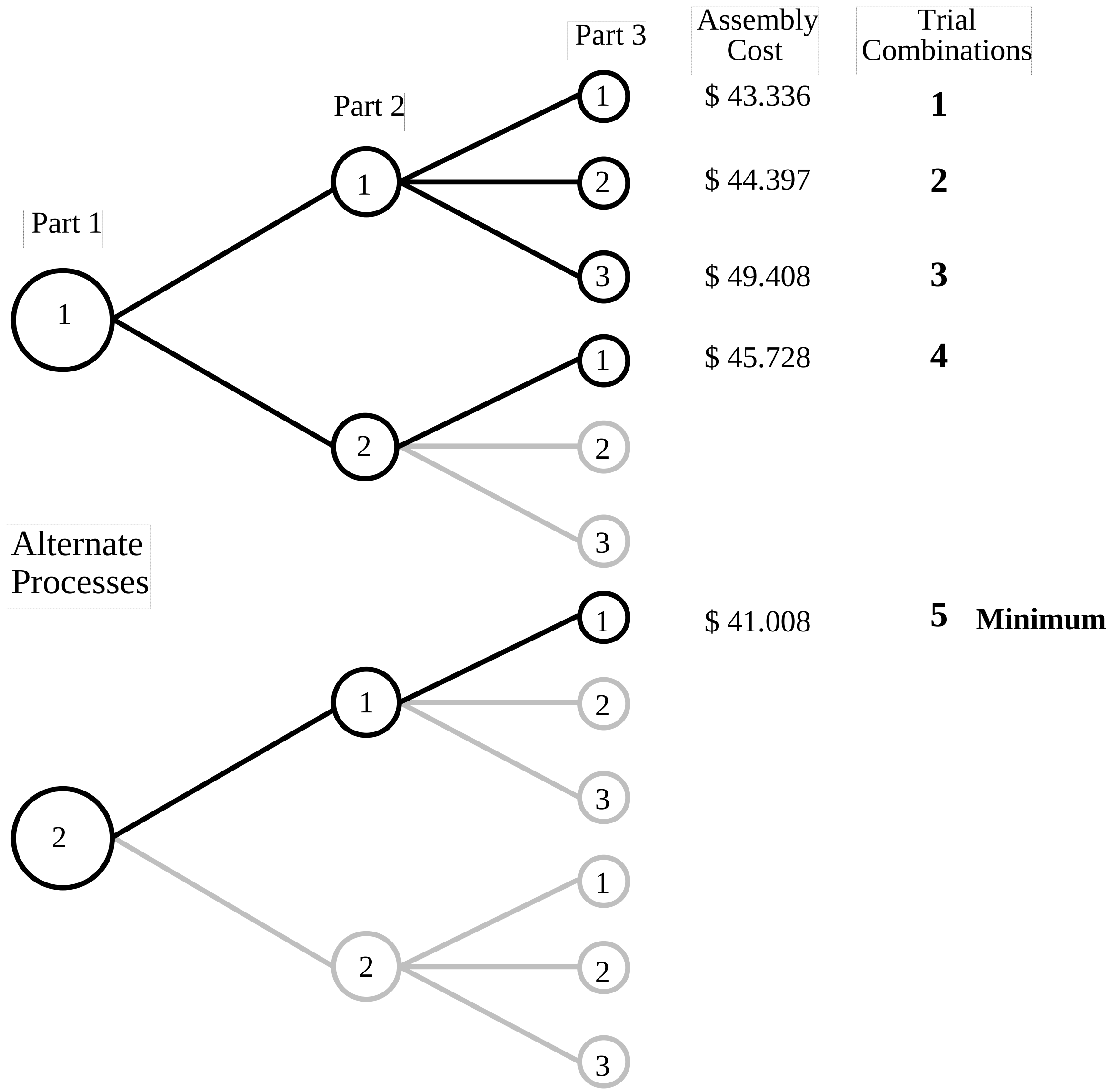


Figure 3.6. Sample Tree for Univariate Search

Before stepping through the part levels of the process curve tree, however, a set of processes that produced a valid or feasible starting case was necessary. Without a valid starting set of processes, no cost comparisons could be made on initial process searches. Therefore, two methods for determining a feasible starting point were developed. First, two levels of the cost curve tree could be searched for a feasible point. Second, the exhaustive search method could be started up until a feasible point was found. If the exhaustive search was started, a tightly constrained problem may end up performing the majority of the exhaustive search before a feasible starting point was found. However, it was critical to the algorithm's functionality that a feasible starting point be found. If none was found, the algorithm could not be used.

For the majority of test cases, the univariate search found the absolute minimum process combination. However, in some cases the univariate method was unable to find the absolute minimum. If the process curves have upper tolerance range constraints, the branch ends are not guaranteed to exhibit a uniform pattern. That means that there could possibly be a large number of local minima throughout the design space. Only a lucky starting place could produce the absolute minimum from the univariate search in such cases Of most concern were process cost functions that had upper and lower tolerance constraints and when process cost curves for the same parts had different exponents. In general, a single univariate search can only guarantee a local minimum.

3.6 Continued Univariate Search

On many test cases, it was discovered that a univariate search would put the solution in the vicinity of the global minimum. The univariate search could then be started from the current univariate optimum and repeated until no better solution was found. The stopping condition for the extended univariate search was met when no process curves were changed during the search. If no process curves were changed, the search could not proceed any further out of the current minimum point that it had found.

Like other general optimization techniques, a linear search is performed until a minimum is found. Then a new search direction is determined and another search is performed until a new search direction cannot be found that will improve on the objective function.

This version of the univariate search was determined to be the best method next to the exhaustive search. It matched the exhaustive search in accuracy in almost every case, and it greatly deceased the number of process combinations tried. Noisy surface effects due to tolerance re-allocation were the principle cause of inability to guarantee a global minimum. Repeating the search nearly eliminated the problem.

A little hindsight revealed that the lowest curve-following method is really very similar to the Univariate Search. Each part level is searched successively for the lowest cost curve. However, the lowest curve-following method does not re-allocate tolerances as it evaluates the curves. Re-allocation is performed only at the end of each univariate search. This made the search surface more noisy and made the method less predictable.

3.7 Applying Tolerance Constraints by End-fixing

Manufacturing processes are generally only capable of producing parts over a specified tolerance range. So an important factor in a valid process selection technique is to ensure that the processes selected are compatible with the allocated tolerances. This was accomplished by specifying the allowed tolerance range for each process. After computing the optimum tolerance allocation for the selected set of processes for an assembly, the resulting tolerances were checked to see if they were within the prescribed range for each process. Those tolerances which were outside the limits were adjusted to the nearest limit.

This "end-fixing" operation caused numerous problems. Whenever a component tolerance was adjusted, the tolerance sum for the assembly was altered, so computation of a new allocation was necessary, while holding the adjusted tolerance fixed. The new allocation sometimes caused other tolerances to pop out-of-bounds, so the end-fixing and re-allocating had to be repeated until no more out-of-bounds tolerances were created. The procedure turned out to be highly order-dependent, giving completely different results depending upon which tolerances were fixed first. It also could result in an infeasible solution when all of the tolerances were fixed and the resulting sum violated the assembly tolerance constraint.

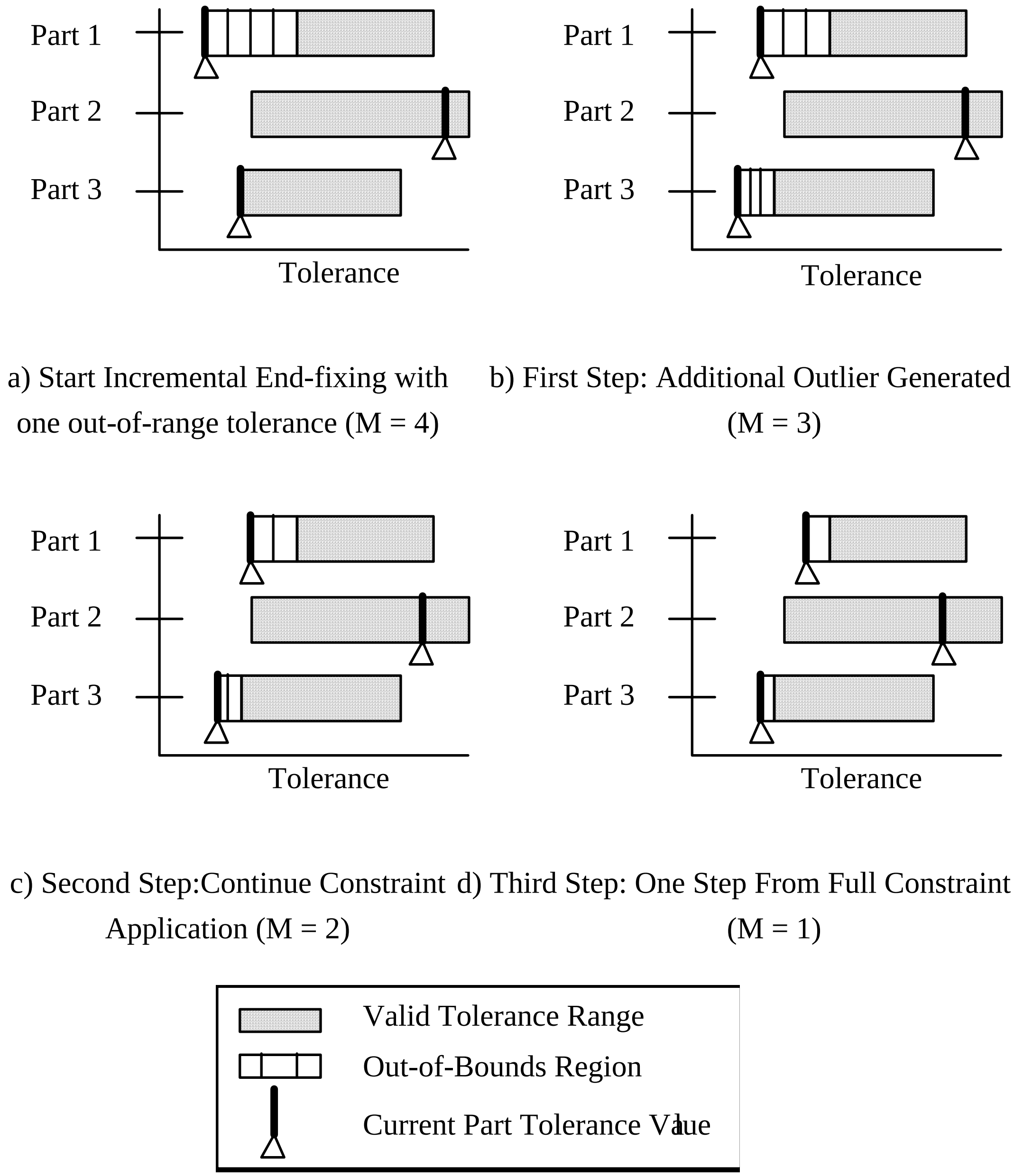
The end-fixing also played havoc with the optimum process search methods. End-fixing produced much greater search surface noise than tolerance re-allocation. None of the search techniques performed well when end-fixing was necessary.

3.8 Incremental End-fixing

Fixing one out-of-bounds tolerance at a time was basically an unstable process. It tended to cause large shifts in the tolerance allocations and unpredictable results. It was decided that a method of gradually applying the tolerance constraints would be more stable. The method for gradually applying the constraints consisted of repeating the tolerance allocation for the selected set of processes by Lagrange multipliers N times and fixing the out-of-range tolerances closer to the process tolerance constraints by steps. The step length was set as the distance from the constraint divided by M as M went from N to

1. By gradually applying the constraints, the unstable effect of large tolerance changes by one-step end-fixing were avoided.

As other tolerances were pushed out of range, they were subjected to the incremental end-fixing at the current iteration of M. If this algorithm was unable to pull the component tolerances inside the constraints to a valid solution, the combination of curves chosen was considered infeasible. N = 4 was found to work well and was used in test problem evaluation. Figure 3.7 below depicts how the incremental end-fixing method pulls the component tolerances into the process constraints.



a) Start Incremental End-fixing with one out-of-range tolerance (M = 4)

b) First Step: Additional Outlier Generated (M = 3)

c) Second Step:Continue Constraint Application (M = 2)

d) Third Step: One Step From Full Constraint (M = 1)

Figure 3.7. Incremental End-fixing

3.9 Process Search Performance with End-fixing

Each of the methods were modified to use the incremental end-fixing algorithm (INC) and tested to see how they were affected by it. After the search algorithms selected a set of processes cost curves, Lagrange multipliers were used to allocate the tolerances and INC was used to assure that process tolerance constraints were met.

3.9.1 Exhaustive Search Method

When out-of-bound tolerances were fixed one at a time at process cost curve limits, the solutions were not always the best available. Also, tightly constrained problems would have a high number of combinations rejected as infeasible. By using INC, the number of infeasible solutions decreased and the optimum costs decreased. Even though the number of tolerance allocations often increased with INC, the increased performance justified increased processing.

3.9.2 Bottom Curve Following Method

This method started with the unconstrained optimum cost solution and only dealt with parts that had their allocated tolerances outside process tolerance constraints. As with the exhaustive search method, the one-step end-fixing concept caused very unpredictable results when tolerance constraints were applied, so INC was implemented to stabilize the end-fixing variations. This method would select the initial set of process cost curves, allocate the tolerances, and apply INC to make them conform to the constraints. Then, it would step through the component tolerance list until it found a tolerance that was fixed to meet the process tolerance constraints. If other processes were available for that component, they were checked for a lower cost by setting each one as the current process and re-allocating the tolerances using INC. If no other process cost curves generated a lower assembly tolerance cost, the original curve was kept. If another process cost curve produced a lower assembly tolerance cost, then the new curve was set and the algorithm started through the tolerance list again.

By using INC the benefits of greater stability and a greater probability of finding a global optimum were realized. This method also kept computing costs to a minimum by only searching the part levels that had to be forced into process tolerance limits. Even though this was a big improvement over the one-step end-fixing method, it was only able to guarantee a local minimum.

3.9.3 Univariate Search Method

The univariate search is dependent on objective function patterns in order to find the optimum value. With one-step end-fixing, the noisy surface would defeat the univariate search sometimes by producing assembly tolerance costs that were not the least cost for the set of process curves selected. However, by using INC, the noisy surface effects were decreased. The assembly tolerance costs produced by INC for a given set of process cost curves seemed to be the least cost set of tolerances with process tolerance constraints applied. After implementing INC in the univariate search, it became almost as accurate as the exhaustive search. The systematic searching of the univariate method coupled with INC for tolerance allocation seemed to be the best combination next to the exhaustive search and INC. Although you still could not guarantee the absolute minimum assembly tolerance cost without running the exhaustive search.

4.0 RESULTS

In order to evaluate the performance of the various methods to find the best process and tolerance combination for minimum assembly tolerance cost, some criteria was set by which all methods were compared. The first comparison was on CPU time needed to produce a solution. System timers were inserted in the computer code to measure the CPU seconds used during an algorithm's execution. Second, the number of combinations tried was recorded to compare the effectiveness of the searching methods. Third, the number of tolerance re-allocations was recorded in order to compare effects of process tolerance ranges. And finally, the results from each method were compared with the results from an exhaustive search. The exhaustive search was always able to find the absolute minimum cost combination; it was used for the standard.

All methods described herein were evaluated on each of the test cases listed in the appendix as well as other intermediate tests for numerical accuracy. Of the many variations tried, it was determined that three methods would be evaluated in detail. The three methods are listed below:

1) Exhaustive Search (EXH)

2) Bottom Curve Follower (BCF)

3) Extended Univariate Search (UNI)

4.1 Unconstrained Problems

The comparison on CPU time is depicted in Figure 4.1 for problems A through I. The resolution of the system timer is 0.01 CPU seconds. For problems A to I, the processes had no tolerance constraints and the exponents on the process cost curves were all set at -1. This was done primarily to produce a uniform set of problems that would compare CPU usage. Also, they correspond closely to the problem set used by Hauglund. The unconstrained processes can, however, represent the general trends associated with increased problem size.

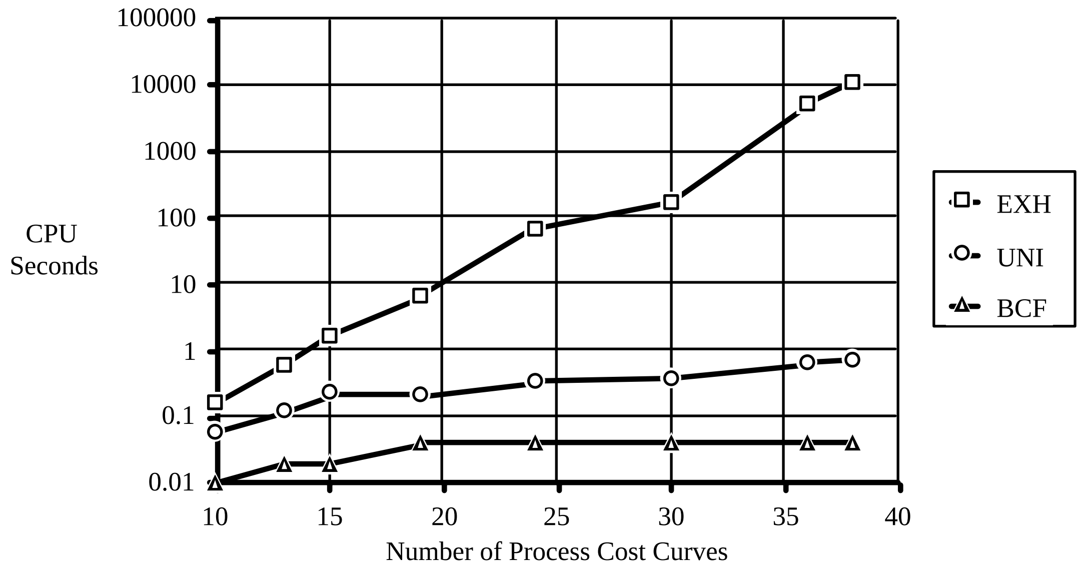


Figure 4.1. CPU Usage Comparison Chart

On problems A to I, each of the three methods evaluated arrived at the correct solution to the problem. However, the CPU usage and number of process combinations tried per problem varied greatly over the different methods. The performance criteria for problems A to I is listed in Table 4.1 below.

Problem		CPU Seconds Used			Combinations Tried			Tol. Re-allocations††		
Name	Size	BCF	UNI	EXH	BCF	UNI	EXH	BCF	UNI	EXH
A	10	0.01	0.06	0.16	2	14	36	1	16	38
B	13	0.02	0.12	0.59	2	16	96	1	18	98
C	15	0.02	0.14	1.24	2	18	192	1	20	194
D	19	0.04	0.21	6.50	2	24	864	1	26	866
F	24	0.04	0.34	67.27	2	26	4096	1	28	4098
G	30	0.04	0.38	175.0	2	48	25600	1	50	25602
H	36	0.04	0.64	5388	2	50	531441	1	52	531443
I	38	0.04	0.73	11616	2	52	1062882	1	54	1062884

†† Numbers show all tolerance re-allocations including those from end-fixing

Table 4.1. Performance Values for Problems A-I

The results from the unbounded problems demonstrate the obvious differences in CPU usage and number of combinations associated with the different methods. Although each of the three methods arrived at the global minimum cost solution, BCF was the least costly method for the unbounded problems. Obviously, EXH used up more CPU time than the other methods, and it increased its CPU usage more rapidly than any of the other methods as the size of the problems increased because all combinations had to be checked. Ideal cases like problems A to I are useful for comparison of search algorithms in unconstrained space, but another more complex set of test cases was necessary to determine performance under more realistic condition in which various process tolerance constraints are applied.

4.2 Problems with Process Constraints

The more complex set of test problems contained various process tolerance constraints as well as having various process cost curve exponent values. For demonstration purposes, three test problems were selected from numerous cases to represent the general behavior of the test methods used. Each of the problems were evaluated under three constraint conditions;

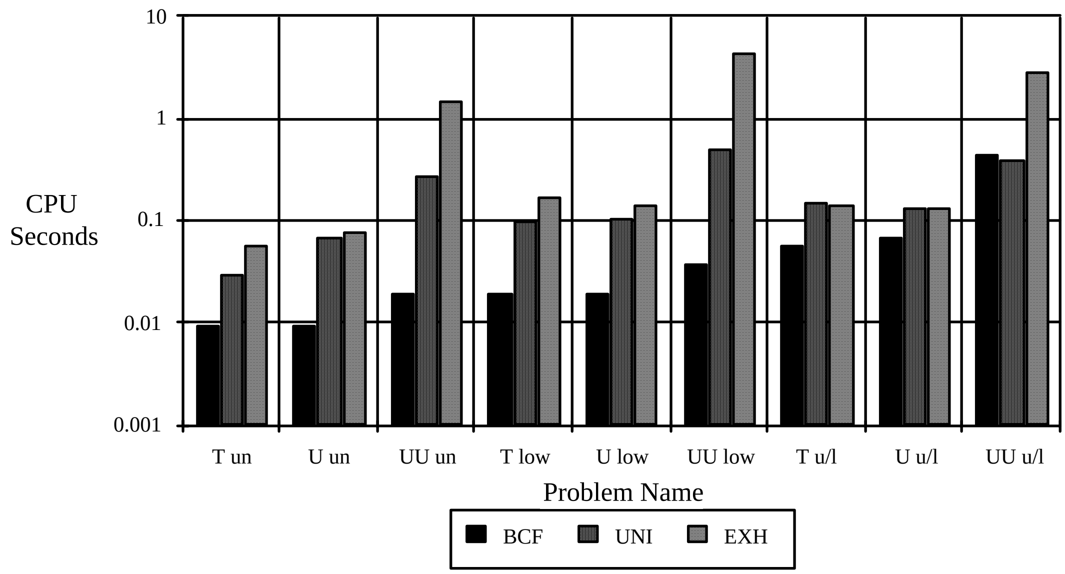
1) upper and lower process tolerance constraints,

2) only lower process tolerance constraints, and

3) unconstrained process tolerance constraints.

In addition to the process tolerance constraints, various combinations of process cost curves were examined. The three problems are labeled T, U, and UU. Problem T had all process cost curve exponents set at -2, problems U and UU had integer cost curve exponents ranging from -1 to -3. The data for problem U was copied twice to create problem UU, and the assembly tolerance for UU was set at twice that of problem U to show the effects of problem size.

Figure 4.2 displays the CPU usage used in the various problems similar to Figure 4.1. Notice how the CPU time decreases as the constraints on the problem are relaxed.



Constraint conditions on processes: un = unbounded tolerance,
low = lower tolerance bounds u/l = upper and lower tolerance bounds

Figure 4.2. CPU Usage for Problems T,U, and UU

The CPU usage is the time needed to return a solution for each of the problems, however, sometimes only a local optimum was returned by the algorithms. In Table 4.2, the results of the tests on problems T, U, and UU are displayed. Note that EXH was the only method to guarantee the absolute minimum assembly tolerance cost in all cases.

Problem		CPU Seconds Used			Combinations Tried			Tol. Re-allocations††		
Name	Size	BCF	UNI	EXH	BCF	UNI	EXH	BCF	UNI	EXH
No Process Tolerance Constraints										
T	7	0.01	0.03	0.06	2	10	12	1	12	15
U	7	0.01*	0.07*	0.08	2	10	12	1	12	14
UU	14	0.02*	0.29	1.50	2	27	144	2	29	176
Only Lower Process Tolerance Constraints										
T	7	0.02	0.10	0.18	2	10	12	1	32	66
U	7	0.02	0.11	0.15	2	10	12	1	32	49
UU	14	0.04	0.51	4.66	2	18	144	2	75	811
Both Upper and Lower Process Tolerance Constraints										
T	7	0.06	0.16	0.15	4	10	12	16	57	76
U	7	0.07*	0.14*	0.14	3	10	12	27	62	74
UU	14	0.47	0.42	2.90	4	18	144	30	115	886

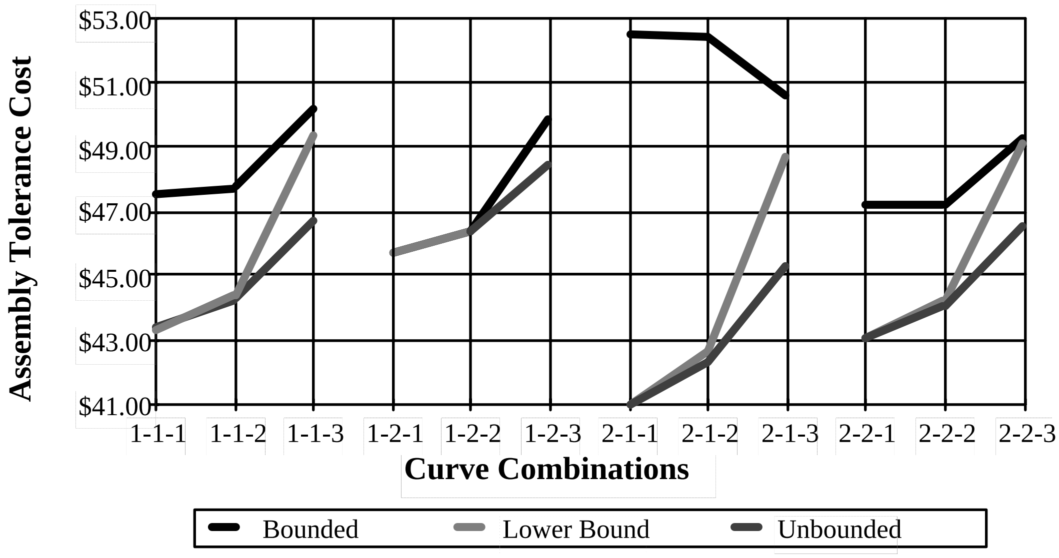
†† Numbers show all tolerance re-allocations including those from end-fixing

* Found a local not global minimum

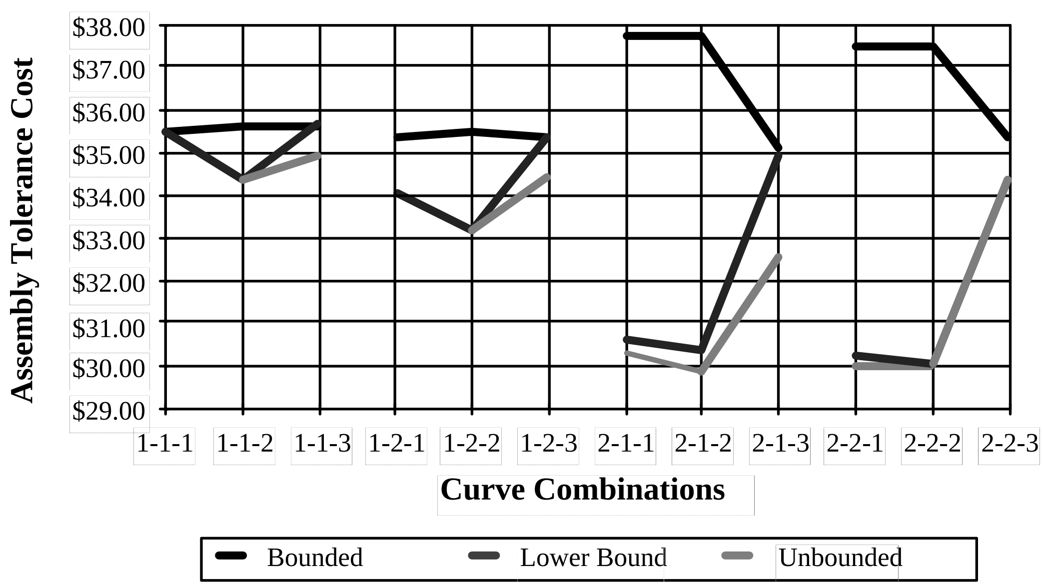
Table 4.2. Performance Data for Problems T,U, and UU

All three methods performed well when the special case of constant cost curve exponents and only lower tolerance constraints were allowed. But the case of mixed cost curve exponents and various tolerance constraints led to only near-minimum solutions. Note that the CPU time does not increase appreciably with addition of constraints, indicating the incremental end-fixing algorithm is efficient. Also, in the results for EXH, the number of re-allocations is much higher than the number of combinations tried. This indicates that a lot of end-fixing was necessary due to allocated tolerances which fell out of the process tolerance ranges.

Figure 4.3 is a plot of assembly tolerance costs for all possible combinations of problems T and U. This gives a feel for the nature of the discontinuous surface the algorithms must search upon. It also shows the patterns of cost increase which occur as the process is discretely varied for each part. The basic shape of the surface is greatly changed when upper tolerance bounds are added. Repeating patterns are disturbed.



a) Problem T Assembly Tolerance Costs



b) Problem U Assembly Tolerance Costs

Figure 4.3. Assembly Tolerance Costs for T and U

4.3 Effects of Problem Size

The interesting results from the complex cases came when the size of the problem increased. In problem UU, UNI was able to find the absolute minimum every time. This result is quite promising because EXH can be run on the smaller problems to guarantee a minimum, and the larger problems that are too CPU intensive for EXH can use UNI. If various starting points for UNI are chosen, the absolute minimum value is almost a surety. It must be remembered, however, that the only way to guarantee the absolute minimum assembly tolerance cost for all cases is to run EXH.

The complete numerical results for problems T, U, and UU can be found in Appendix I. The appendix tables list the combinations and resulting costs for EXH. In parallel columns, the search order of UNI is also included. Each UNI search is shown as a separate column. Each UNI search starts at the minimum value of the previous search. The absolute minimum is marked to show the desired answer. Note how UNI found the optimum usually in two searches except for Problem U where it only found a local minimum. The unconstrained process solution to UU did require three searches, but UNI was still able to find the solution in a fraction of the time EXH needed. In some cases identical minimum costs occur at more than one process combinations, then UNI finds only one of them. The minimum found depends on the starting point of the search.

4.4 Limitations

The biggest concerns for cost optimization are 1) when alternate process cost curves do not have overlapping tolerance ranges because of upper and lower tolerance constraints, and 2) when process cost curves have different exponents. Both provide complexities in themselves and multiply difficulties if combined.

If process cost curves have upper and lower tolerance constraints, the efficient algorithms have no stable method of solution. All three of the solution methods return a local minimum, but the absolute minimum is only guaranteed by EXH. Using only lower tolerance constraints on the process cost curves did not cause significant problems and is actually more true to life. Most manufacturing processes have limitations on the tightness of their tolerances, but they are not limited on how rough they can go. Also, when the tolerances get rougher, other less costly options become available and would provide a lower process cost curve.

The mixed cost curve exponents were as difficult if not worse than the tolerance constraints. However, UNI found the absolute minimum on almost all of the test problems. If different starting points were chosen, UNI became even more likely to find the solution.

Problem size had a big influence on the ability of the methods to find the absolute minimum cost process combination. UNI had the most trouble with small size problems. But if the problem was small, EXH was about the same cost as UNI and should be used. When the problems got very large, UNI found the solution every time.

5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The main conclusion from all tests performed is that EXH is the only method to guarantee the global minimum assembly tolerance cost. The main problem associated with the process selection was the noisy search surface. With multiple minima scattered over the search surface, most methods will only find a local minimum and stay there. That is why a global optimum is so difficult to guarantee when searching over a noisy surface.

The decision on which method to use for analysis can be based on the number of process combinations that exist in a problem. In general, EXH should be used when the number of combinations is less than fifty. When the number of EXH combinations will exceed fifty, UNI is recommended. Of course if the computing resources are unlimited, EXH will guarantee the global optimum value for any problem.

Other factors may also be considered to influence the decision on which process selection method to use.

1) <u>Upper and lower process tolerance constraints used on process cost curves.</u>
   When this condition arises, the only guaranteed solution method is EXH. However, if the number of EXH combinations will exceed fifty or the time necessary for solution will exceed computer allocation time, then UNI should be used.

2) <u>Only lower process tolerance constraints are used on process cost curves.</u>
   Here UNI is recommended for all problems except for those with the number of EXH combinations less than twenty or thirty. The exact cut-off point is problem dependent and would require further study.

3) <u>No process tolerance constraints are used on process cost curves.</u>
   When this condition exists, all methods in this thesis have a high probability of returning the absolute minimum for the problem. The only restriction arises when the process cost curve exponents are not the same. If the curve exponents are the same for all processes connected to one part, but differ from part to part, BCF, UNI or EXH can be used. If the process cost curve exponents vary within the processes of one part, then EXH should be used

until the number of combinations exceed fifty, and then a UNI search is recommended. However, if many tolerances need to be fixed for the solution, the rules mentioned in (2) should be followed.

The limitations on using the methods other than EXH may raise some concerns about the usefulness of the techniques presented here. However, the search surface associated with the discrete process selection case can become something like a random number field. The global minimum may be hidden amongst maximum extremes and/or multitudes of local minima. If extreme conditions exist, extreme measures must be taken to solve the problem. Therefore, EXH should always be available as an option.

5.2 Recommendations for Other Possible Methods

Although the methods presented herein are useful and can solve many complex cases, they do not rule out other possibilities. A better method to solve the case where upper and lower process tolerance constraints are applied may need to be developed. Although the need for upper bounds has not been found from the limited data available, some specialty applications may have circumstances that warrant such a method. Exhaustive search times increase almost exponentially as the number of parts and processes go up. Therefore, a method for locating the global minimum on a noisy surface, while still maintaining the process tolerance constraints would be of great use.

5.2.1 Simulated Annealing Possibility

One documented method for solving multiple minima combinatorial problems is the Simulated Annealing method. It is modelled after the ordering of atoms into a low-energy state in a material as it is annealed.

Bohachevsky[5] and Kirkpatrick[6] along with others have demonstrated how exhaustive searches are unnecessary for many large combinatorial problems if the problems can be formulated into an annealing problem. The basic method randomly perturbs the system parameters on a trial basis. If the objective function improves, the perturbations are made permanent. If the objective function gets worse, it will still keep the perturbations if there is a probability that by doing so it will eventually find an extreme point better than the current one. This ability to jump out of local minima can lead to the global minimum being found. However, it cannot guarantee the global optimum. When the number of exhaustive search combinations becomes extremely large, this may be a viable alternative. No references were discovered applying such an algorithm to a process selection problem for tolerance analysis, but other combinatorial examples tend to encourage further investigation.

5.2.2 Gradient Possibility

A major consideration for this problem was to use the Branch and Bound algorithm in OPTDES.BYU to solve this type of problem with combined continuous and discrete variables. However, the method used in OPTDES to deal with discrete solutions was designed for a different type of problem than that needed for the specific problem of multiple process cost curve analysis. Therefore, it was deemed inappropriate for the cost–tolerance problem in its present form. However, the cost curve parameters were used in OPTDES to generate a contour plot of the assembly tolerance cost contours as a function of the cost curve parameters. (see Figure 3.2) If some general combined gradient method could be developed to take advantage of the cost gradients and still account for tolerance range limitations, it may prove successful in dealing with the exceptional cases that only the exhaustive method has been able to solve reliably.

5.3 Need for Good Data

The main restriction on using any of the process selection methods is the need for accurate process cost data. Once the data is established, analysis may begin. However, exact costs for various processes may not be readily available in published form. Often it is only available as a "rule of thumb" of some experienced manufacturing system analyst.

One project currently under way is the gathering of data at Garrett Turbine Engine Co. on manufacturing costs. They determined that many parts have the processes prescribed before manufacturing begins, but the number of cuts and type of cuts are not determined until the part reaches the production floor. They are collecting data on similar processes with rough and finish cuts coupled with grinding or finish processing. If sufficient data can be gathered, it may be possible to break down a process into detailed cost curves similar to Figure 5.1. Then the methods presented in this thesis could be used to analyze the assembly and find the least cost solution.

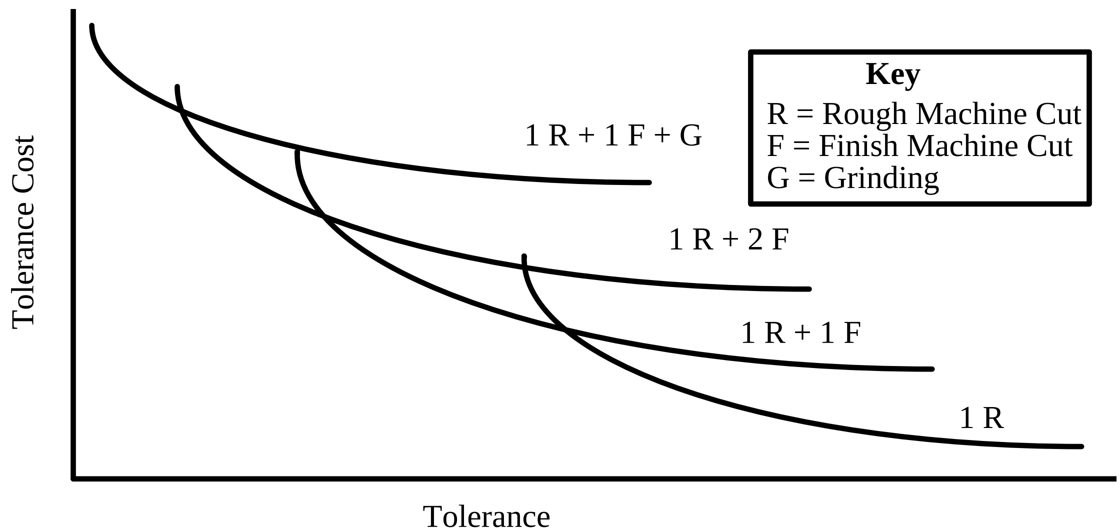


Figure 5.1. Example Combinations of Processes

By breaking down the processes performed into different operations, a preliminary manufacturing process plan can be generated before the design leaves the engineer's hands. This is the type of communication that needs to be conveyed from the shop floor to the engineering designers. When process costs and capabilities are known during the design phases of a project, increased cost savings can and will be achieved.

5.4 CATS.BYU Recommendations

The tests performed in development of this thesis have demonstrated the application of process selection in tolerance analysis and allocation. Because of the varied nature in current tolerance analysis problems, this type of problem solving should be included in the CATS.BYU program. This would bring increased capability for design for manufacture to CATS. Although each of the methods described above have merit, only the EXH and UNI methods are to be considered as useful and flexible methods for process selection. Both EXH and UNI are therefore recommended to be implemented in the CATS.BYU program. Because of the compact nature of BCF, it may also want to be included for special cases, but it is not considered sufficiently flexible as a design method to require implementation.

In order to provide a useful interface for implementation of these methods the following suggestions are warranted.

5.4.1 Reference Handbook Data

As mentioned above, the most critical part of tolerance analysis is having accurate data. Although engineers could look up cost data in reference tables or handbooks, inclusion of cost curves for several standard processes as on-line data would enhance the usefulness of process allocation techniques. Perhaps process curve data could be retrieved through structured questions or on-screen selection of existing process models. Of course the ability to enter user-specified cost data must be included, as should be the ability to add and update data in the reference database.

Care should be taken as the methods for data retrieval are implemented with concentration on presenting possibilities in a manner most acceptable to the user. Processes could be selected by themselves, but the selection could be guided by type of feature desired or type of material to be used or even by reference to what is most commonly done in the shop by having a crude artificial intelligence for assistance.

5.4.2 Flexible Control Features

Besides flexible data access, other flexible features can enhance analysis functionality and encourage greater use. Decisions on process selection methods may be preset according to problem size and complexity. For example, if the number of combinations is less than 20, the system would automatically use EXH. The user may also want to analyze specific process combinations with and/or without process tolerance constraints. Tolerance constraints might be turned on or off for analysis. An individual process could be flagged as "fixed", turning off the search for alternate processes for a specific part. And for the extreme cases, the user should be allowed to select a specific combination of processes as a starting point for the process search. Another helpful feature would keep track of the five lowest cost process combinations. Then if the user wanted, they could decide between several good selections based on other considerations than simply lowest cost. By including flexibility with CATS.BYU, its use as an engineering design tool will increase and allow it to grow and change as as engineering moves into the modern manufacturing era.

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APPENDIX I

Data From Exhaustive and Extended Univariate Solutions

Problem T: Exhaustive Search Iterations

(Univariate Search Iterations Listed)

Exhaust. Iteration	Problem T Unbounded Asm. Tol. Cost	Curve Comb.			Search 1	Search 2	Min
1	43.3364	1	1	1	1	5	
2	44.2613	1	1	2	2		
3	46.7066	1	1	3	3		
4	45.7279	1	2	1	4		min
5	46.3753	1	2	2			
6	48.4215	1	2	3			
7	41.0082	2	1	1	5	1	
8	42.3019	2	1	2		2	
9	45.2759	2	1	3		3	
10	43.0534	2	2	1		4	
11	44.0522	2	2	2			
12	46.6035	2	2	3			
	41.0082	Minimum Cost					

Exhaust. Iteration	Problem T Lower Bounds Asm. Tol. Cost	Curve Comb.			Search 1	Search 2	Min
1	43.3364	1	1	1	1	5	
2	44.3970	1	1	2	2		
3	49.4079	1	1	3	3		
4	45.7279	1	2	1	4		
5	46.4066	1	2	2			
6	49.9158	1	2	3			
7	41.0082	2	1	1	5	1	min
8	42.6849	2	1	2		2	
9	48.7890	2	1	3		3	
10	43.0534	2	2	1		4	
11	44.2276	2	2	2			
12	49.1018	2	2	3			
	41.0082	Minimum Cost					

Exhaust. Iteration	Prob. T Upper & Lower Bounds Asm. Tol. Cost	Curve Comb.			Search 1	Search 2	Min
1	47.5672	1	1	1	1	4	
2	47.7614	1	1	2	2		
3	50.1881	1	1	3	3		
4	45.7279	1	2	1	4	1	
5	46.4066	1	2	2		2	
6	49.9158	1	2	3		3	
7	52.5071	2	1	1			
8	52.4271	2	1	2			
9	50.5144	2	1	3			
10	47.2084	2	2	1	5	5	
11	47.2078	2	2	2			
12	49.3269	2	2	3			
	45.7279	Minimum Cost					

Problem U: Exhaustive Search Iterations

(Univariate Search Iterations Listed)

Exhaust. Iteration	Problem U Unbounded Asm. Tol. Cost	Curve Comb.			Search 1	Search 2	Min
1	35.5269	1	1	1	1		
2	34.3769	1	1	2	2		
3	34.9061	1	1	3	3		
4	34.1044	1	2	1		5	
5	33.1808	1	2	2	4		
6	34.4884	1	2	3			
7	30.3059	2	1	1		4	
8	29.8445	2	1	2			
9	32.5651	2	1	3			
10	29.9856	2	2	1		2	
11	30.0522	2	2	2	5	1	
12	34.3689	2	2	3		3	
	29.8445	Minimum Value					

Exhaust. Iteration	Problem U Lower Bounds Asm. Tol. Cost	Curve Comb.			Search 1	Search 2	Min
1	35.5269	1	1	1	1		
2	34.3769	1	1	2	2		
3	35.6459	1	1	3	3		
4	34.1044	1	2	1			
5	33.1808	1	2	2	4	5	
6	35.3808	1	2	3			
7	30.6255	2	1	1		2	
8	30.3617	2	1	2		4	
9	34.9330	2	1	3			
10	30.2420	2	2	1			
11	30.0522	2	2	2	5	1	min
12	34.3746	2	2	3		3	
	30.0522	Minimum Value					

Exhaust. Iteration	Prob. U Upper & Lower Bounds Asm. Tol. Cost	Curve Comb.			Search 1	Search 2	Min
1	35.5269	1	1	1	1	4	
2	35.6147	1	1	2	2		
3	35.6459	1	1	3	3		
4	35.3446	1	2	1	4	1	
5	35.4918	1	2	2		2	
6	35.3899	1	2	3		3	
7	37.7348	2	1	1			
8	37.7348	2	1	2			
9	35.0931	2	1	3			
10	37.4787	2	2	1	5	5	
11	37.4787	2	2	2			
12	35.3096	2	2	3			
	35.0931	Minimum Value					

Problem UU Without Tolerance Bounds: Exhaustive Search Iterations

(Univariate Search Iterations Listed)

Problem UU Unbounded

Iter	Tol Cost	Process Curve Combinations						Search 1	Search 2	Search 3	Min
1	71.05	1	1	1	1	1	1	1			
2	69.87	1	1	1	1	1	2	2			
3	69.96	1	1	1	1	1	3	3			
4	69.56	1	1	1	1	2	1				
5	68.47	1	1	1	1	2	2	4			
6	68.84	1	1	1	1	2	3				
7	65.32	1	1	1	2	1	1				
8	64.38	1	1	1	2	1	2				
9	65.21	1	1	1	2	1	3				
10	64.20	1	1	1	2	2	1				
11	63.43	1	1	1	2	2	2	5			
12	64.67	1	1	1	2	2	3				
13	69.87	1	1	2	1	1	1				
14	68.75	1	1	2	1	1	2				
15	69.02	1	1	2	1	1	3				
16	68.47	1	1	2	1	2	1				
17	67.46	1	1	2	1	2	2				
18	68.05	1	1	2	1	2	3				
19	64.38	1	1	2	2	1	1				
20	63.55	1	1	2	2	1	2				
21	64.65	1	1	2	2	1	3				
22	63.43	1	1	2	2	2	1				
23	62.77	1	1	2	2	2	2	6			
24	64.33	1	1	2	2	2	3				
25	69.96	1	1	3	1	1	1				
26	69.02	1	1	3	1	1	2				
27	69.81	1	1	3	1	1	3				
28	68.84	1	1	3	1	2	1				
29	68.05	1	1	3	1	2	2				
30	69.23	1	1	3	1	2	3				
31	65.21	1	1	3	2	1	1				
32	64.65	1	1	3	2	1	2				
33	66.44	1	1	3	2	1	3				
34	64.67	1	1	3	2	2	1				
35	64.33	1	1	3	2	2	2	7			
36	66.69	1	1	3	2	2	3				
37	69.56	1	2	1	1	1	1				
38	68.47	1	2	1	1	1	2				
39	68.84	1	2	1	1	1	3				
40	68.21	1	2	1	1	2	1				
41	67.24	1	2	1	1	2	2				
42	67.94	1	2	1	1	2	3				
43	64.20	1	2	1	2	1	1				
44	63.43	1	2	1	2	1	2				
45	64.67	1	2	1	2	1	3				
46	63.34	1	2	1	2	2	1				
47	62.74	1	2	1	2	2	2				
48	64.47	1	2	1	2	2	3				

Problem UU Unbounded

Iter	Tol Cost	Process Curve Combinations						Search 1	Search 2	Search 3	Min
49	68.47	1	2	2	1	1	1				
50	67.46	1	2	2	1	1	2				
51	68.05	1	2	2	1	1	3				
52	67.24	1	2	2	1	2	1				
53	66.36	1	2	2	1	2	2				
54	67.33	1	2	2	1	2	3				
55	63.43	1	2	2	2	1	1		9		
56	62.77	1	2	2	2	1	2			9	
57	64.33	1	2	2	2	1	3				
58	62.74	1	2	2	2	2	1				
59	62.29	1	2	2	2	2	2	8			
60	64.39	1	2	2	2	2	3				
61	68.84	1	2	3	1	1	1				
62	68.05	1	2	3	1	1	2				
63	69.23	1	2	3	1	1	3				
64	67.94	1	2	3	1	2	1				
65	67.33	1	2	3	1	2	2				
66	68.98	1	2	3	1	2	3				
67	64.67	1	2	3	2	1	1				
68	64.33	1	2	3	2	1	2				
69	66.69	1	2	3	2	1	3				
70	64.47	1	2	3	2	2	1				
71	64.39	1	2	3	2	2	2				
72	67.42	1	2	3	2	2	3				
73	65.32	2	1	1	1	1	1				
74	64.38	2	1	1	1	1	2				
75	65.21	2	1	1	1	1	3				
76	64.20	2	1	1	1	2	1				
77	63.43	2	1	1	1	2	2				
78	64.67	2	1	1	1	2	3				
79	60.61	2	1	1	2	1	1				
80	60.08	2	1	1	2	1	2				
81	61.95	2	1	1	2	1	3				
82	60.12	2	1	1	2	2	1				
83	59.81	2	1	1	2	2	2				
84	62.27	2	1	1	2	2	3				
85	64.38	2	1	2	1	1	1				
86	63.55	2	1	2	1	1	2				
87	64.65	2	1	2	1	1	3				
88	63.43	2	1	2	1	2	1				
89	62.77	2	1	2	1	2	2				
90	64.33	2	1	2	1	2	3				
91	60.08	2	1	2	2	1	1		8		
92	59.69	2	1	2	2	1	2			8	
93	61.94	2	1	2	2	1	3				
94	59.81	2	1	2	2	2	1				
95	59.67	2	1	2	2	2	2				min
96	62.58	2	1	2	2	2	3				

Problem UU Unbounded

Iter.	Tol Cost	Process Curve Combinations						Search 1	Search 2	Search 3	Min
97	65.21	2	1	3	1	1	1				
98	64.65	2	1	3	1	1	2				
99	66.44	2	1	3	1	1	3				
100	64.67	2	1	3	1	2	1				
101	64.33	2	1	3	1	2	2				
102	66.69	2	1	3	1	2	3				
103	61.95	2	1	3	2	1	1				
104	61.94	2	1	3	2	1	2				
105	65.13	2	1	3	2	1	3				
106	62.27	2	1	3	2	2	1				
107	62.58	2	1	3	2	2	2				
108	66.57	2	1	3	2	2	3				
109	64.20	2	2	1	1	1	1				
110	63.43	2	2	1	1	1	2				
111	64.67	2	2	1	1	1	3				
112	63.34	2	2	1	1	2	1				
113	62.74	2	2	1	1	2	2				
114	64.47	2	2	1	1	2	3				
115	60.12	2	2	1	2	1	1		6		
116	59.81	2	2	1	2	1	2			6	
117	62.27	2	2	1	2	1	3				
118	59.97	2	2	1	2	2	1				
119	59.93	2	2	1	2	2	2				
120	63.09	2	2	1	2	2	3				
121	63.43	2	2	2	1	1	1		5		
122	62.77	2	2	2	1	1	2			5	
123	64.33	2	2	2	1	1	3				
124	62.74	2	2	2	1	2	1				
125	62.29	2	2	2	1	2	2				
126	64.39	2	2	2	1	2	3				
127	59.81	2	2	2	2	1	1		4	1	
128	59.67	2	2	2	2	1	2			2	min
129	62.58	2	2	2	2	1	3			3	
130	59.93	2	2	2	2	2	1		2		
131	60.10	2	2	2	2	2	2	9	1	4	
132	63.79	2	2	2	2	2	3		3		
133	64.67	2	2	3	1	1	1				
134	64.33	2	2	3	1	1	2				
135	66.69	2	2	3	1	1	3				
136	64.47	2	2	3	1	2	1				
137	64.39	2	2	3	1	2	2				
138	67.42	2	2	3	1	2	3				
139	62.27	2	2	3	2	1	1		7		
140	62.58	2	2	3	2	1	2			7	
141	66.57	2	2	3	2	1	3				
142	63.09	2	2	3	2	2	1				
143	63.79	2	2	3	2	2	2				
144	68.74	2	2	3	2	2	3				

Problem UU With Only Lower Bounds: Exhaustive Search Iterations

(Univariate Search Iterations Listed)

	Problem UU Lower Bounds									
	Tol	Process Curve						Search		
Iter	Cost	Combinations						1	2	Min
1	71.05	1	1	1	1	1	1	1		
2	69.87	1	1	1	1	1	2	2		
3	69.96	1	1	1	1	1	3	3		
4	69.56	1	1	1	1	2	1			
5	68.47	1	1	1	1	2	2	4		
6	68.86	1	1	1	1	2	3			
7	65.32	1	1	1	2	1	1			
8	64.42	1	1	1	2	1	2			
9	66.08	1	1	1	2	1	3			
10	64.24	1	1	1	2	2	1			
11	63.59	1	1	1	2	2	2	5		
12	65.77	1	1	1	2	2	3			
13	69.87	1	1	2	1	1	1			
14	68.75	1	1	2	1	1	2			
15	69.05	1	1	2	1	1	3			
16	68.47	1	1	2	1	2	1			
17	67.46	1	1	2	1	2	2			
18	68.22	1	1	2	1	2	3			
19	64.42	1	1	2	2	1	1			
20	63.72	1	1	2	2	1	2			
21	65.90	1	1	2	2	1	3			
22	63.59	1	1	2	2	2	1			
23	63.09	1	1	2	2	2	2	6		
24	65.62	1	1	2	2	2	3			
25	69.96	1	1	3	1	1	1			
26	69.05	1	1	3	1	1	2			
27	71.29	1	1	3	1	1	3			
28	68.86	1	1	3	1	2	1			
29	68.22	1	1	3	1	2	2			
30	71.02	1	1	3	1	2	3			
31	66.08	1	1	3	2	1	1			
32	65.90	1	1	3	2	1	2			
33	70.45	1	1	3	2	1	3			
34	65.77	1	1	3	2	2	1			
35	65.62	1	1	3	2	2	2	7		
36	69.97	1	1	3	2	2	3			
37	69.56	1	2	1	1	1	1			
38	68.47	1	2	1	1	1	2			
39	68.86	1	2	1	1	1	3			
40	68.21	1	2	1	1	2	1			
41	67.24	1	2	1	1	2	2			
42	68.10	1	2	1	1	2	3			
43	64.24	1	2	1	2	1	1			
44	63.59	1	2	1	2	1	2			
45	65.77	1	2	1	2	1	3			
46	63.46	1	2	1	2	2	1			
47	62.98	1	2	1	2	2	2			
48	65.48	1	2	1	2	2	3			

	Problem UU Lower Bounds									
	Tol	Process Curve						Search		
Iter	Cost	Combinations						1	2	Min
49	68.47	1	2	2	1	1	1			
50	67.46	1	2	2	1	1	2			
51	68.22	1	2	2	1	1	3			
52	67.24	1	2	2	1	2	1			
53	66.36	1	2	2	1	2	2			
54	67.68	1	2	2	1	2	3			
55	63.59	1	2	2	2	1	1			
56	63.09	1	2	2	2	1	2			
57	65.62	1	2	2	2	1	3			
58	62.98	1	2	2	2	2	1			
59	62.58	1	2	2	2	2	2	8	9	
60	65.35	1	2	2	2	2	3			
61	68.86	1	2	3	1	1	1			
62	68.22	1	2	3	1	1	2			
63	71.02	1	2	3	1	1	3			
64	68.10	1	2	3	1	2	1			
65	67.68	1	2	3	1	2	2			
66	70.76	1	2	3	1	2	3			
67	65.77	1	2	3	2	1	1			
68	65.62	1	2	3	2	1	2			
69	69.97	1	2	3	2	1	3			
70	65.48	1	2	3	2	2	1			
71	65.35	1	2	3	2	2	2			
72	69.61	1	2	3	2	2	3			
73	65.32	2	1	1	1	1	1			
74	64.42	2	1	1	1	1	2			
75	66.08	2	1	1	1	1	3			
76	64.24	2	1	1	1	2	1			
77	63.59	2	1	1	1	2	2			
78	65.77	2	1	1	1	2	3			
79	61.25	2	1	1	2	1	1			
80	60.96	2	1	1	2	1	2			
81	64.35	2	1	1	2	1	3			
82	60.84	2	1	1	2	2	1			
83	60.60	2	1	1	2	2	2			
84	64.08	2	1	1	2	2	3			
85	64.42	2	1	2	1	1	1			
86	63.72	2	1	2	1	1	2			
87	65.90	2	1	2	1	1	3			
88	63.59	2	1	2	1	2	1			
89	63.09	2	1	2	1	2	2			
90	65.62	2	1	2	1	2	3			
91	60.96	2	1	2	2	1	1			
92	60.72	2	1	2	2	1	2			
93	64.32	2	1	2	2	1	3			
94	60.60	2	1	2	2	2	1			
95	60.41	2	1	2	2	2	2		8	
96	64.05	2	1	2	2	2	3			

	Problem UU Lower Bounds									
	Tol	Process Curve						Search		
Iter.	Cost	Combinations						1	2	Min
97	66.08	2	1	3	1	1	1			
98	65.90	2	1	3	1	1	2			
99	70.45	2	1	3	1	1	3			
100	65.77	2	1	3	1	2	1			
101	65.62	2	1	3	1	2	2			
102	69.97	2	1	3	1	2	3			
103	64.35	2	1	3	2	1	1			
104	64.32	2	1	3	2	1	2			
105	69.87	2	1	3	2	1	3			
106	64.08	2	1	3	2	2	1			
107	64.05	2	1	3	2	2	2			
108	69.23	2	1	3	2	2	3			
109	64.24	2	2	1	1	1	1			
110	63.59	2	2	1	1	1	2			
111	65.77	2	2	1	1	1	3			
112	63.46	2	2	1	1	2	1			
113	62.98	2	2	1	1	2	2			
114	65.48	2	2	1	1	2	3			
115	60.84	2	2	1	2	1	1			
116	60.60	2	2	1	2	1	2			
117	64.08	2	2	1	2	1	3			
118	60.48	2	2	1	2	2	1			
119	60.28	2	2	1	2	2	2		6	
120	63.82	2	2	1	2	2	3			
121	63.59	2	2	2	1	1	1			
122	63.09	2	2	2	1	1	2			
123	65.62	2	2	2	1	1	3			
124	62.98	2	2	2	1	2	1			
125	62.58	2	2	2	1	2	2		5	
126	65.35	2	2	2	1	2	3			
127	60.60	2	2	2	2	1	1			
128	60.41	2	2	2	2	1	2		4	
129	64.05	2	2	2	2	1	3			
130	60.28	2	2	2	2	2	1		2	
131	60.10	2	2	2	2	2	2	9	1	min
132	63.79	2	2	2	2	2	3		3	
133	65.77	2	2	3	1	1	1			
134	65.62	2	2	3	1	1	2			
135	69.97	2	2	3	1	1	3			
136	65.48	2	2	3	1	2	1			
137	65.35	2	2	3	1	2	2			
138	69.61	2	2	3	1	2	3			
139	64.08	2	2	3	2	1	1			
140	64.05	2	2	3	2	1	2			
141	69.23	2	2	3	2	1	3			
142	63.82	2	2	3	2	2	1			
143	63.79	2	2	3	2	2	2		7	
144	68.75	2	2	3	2	2	3			

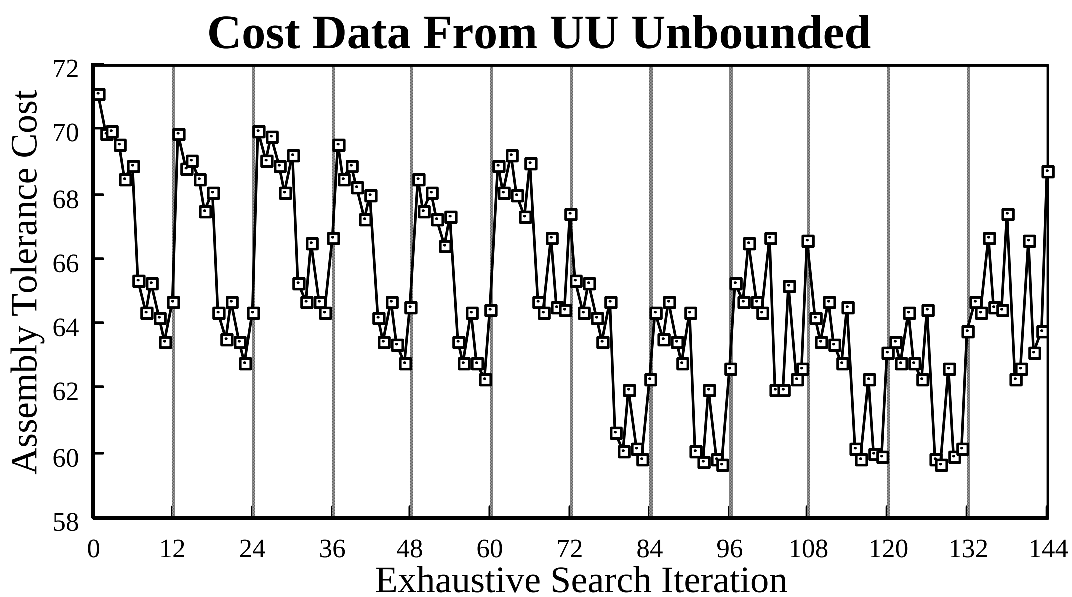
Problem UU With Upper and Lower Bounds: Exhaustive Search Iterations

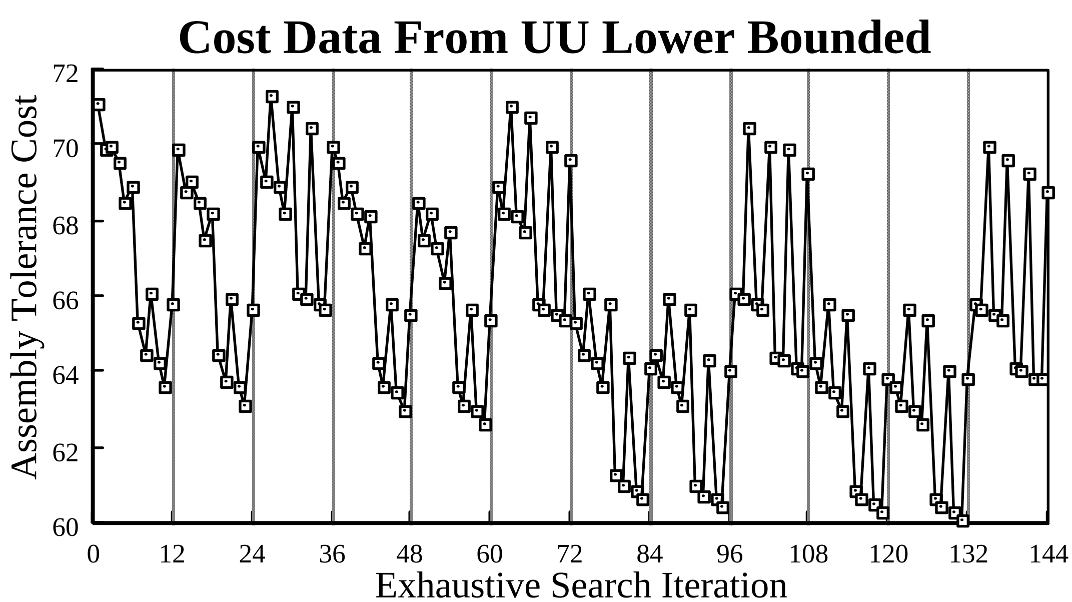
(Univariate Search Iterations Listed)

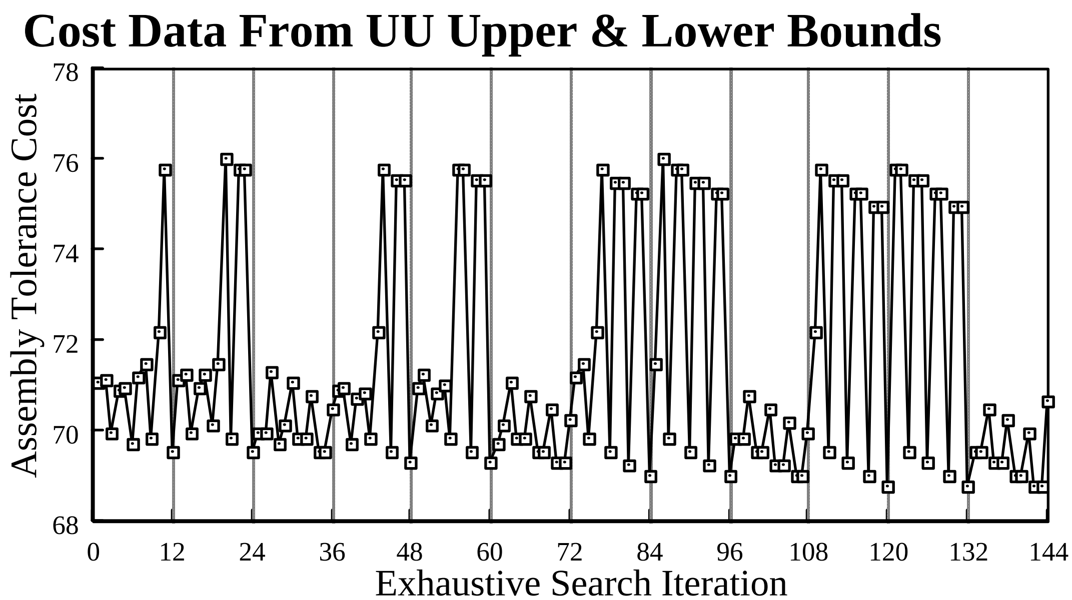
	Prob UU Upper & Lower Bounds									
	Tol	Process Curve						Search		
Iter	Cost	Combinations						1	2	Min
1	71.05	1	1	1	1	1	1	1		
2	71.13	1	1	1	1	1	2	2		
3	69.96	1	1	1	1	1	3	3		
4	70.86	1	1	1	1	2	1			
5	70.96	1	1	1	1	2	2			
6	69.71	1	1	1	1	2	3	4		
7	71.17	1	1	1	2	1	1			
8	71.46	1	1	1	2	1	2			
9	69.80	1	1	1	2	1	3			
10	72.16	1	1	1	2	2	1			
11	75.77	1	1	1	2	2	2			
12	69.54	1	1	1	2	2	3	5		
13	71.13	1	1	2	1	1	1			
14	71.23	1	1	2	1	1	2			
15	69.97	1	1	2	1	1	3			
16	70.96	1	1	2	1	2	1			
17	71.24	1	1	2	1	2	2			
18	70.10	1	1	2	1	2	3			
19	71.46	1	1	2	2	1	1			
20	76.02	1	1	2	2	1	2			
21	69.80	1	1	2	2	1	3			
22	75.77	1	1	2	2	2	1			
23	75.77	1	1	2	2	2	2			
24	69.54	1	1	2	2	2	3	6		
25	69.96	1	1	3	1	1	1			
26	69.97	1	1	3	1	1	2			
27	71.29	1	1	3	1	1	3			
28	69.71	1	1	3	1	2	1			
29	70.10	1	1	3	1	2	2			
30	71.04	1	1	3	1	2	3			
31	69.80	1	1	3	2	1	1			
32	69.80	1	1	3	2	1	2			
33	70.74	1	1	3	2	1	3			
34	69.54	1	1	3	2	2	1			
35	69.54	1	1	3	2	2	2			
36	70.48	1	1	3	2	2	3	7		
37	70.86	1	2	1	1	1	1			
38	70.96	1	2	1	1	1	2			
39	69.71	1	2	1	1	1	3			
40	70.69	1	2	1	1	2	1			
41	70.82	1	2	1	1	2	2			
42	69.84	1	2	1	1	2	3			
43	72.16	1	2	1	2	1	1			
44	75.77	1	2	1	2	1	2			
45	69.54	1	2	1	2	1	3			
46	75.51	1	2	1	2	2	1			
47	75.51	1	2	1	2	2	2			
48	69.29	1	2	1	2	2	3	8	9	

	Prob UU Upper & Lower Bounds									
	Tol	Process Curve						Search		
Iter	Cost	Combinations						1	2	Min
49	70.96	1	2	2	1	1	1			
50	71.24	1	2	2	1	1	2			
51	70.10	1	2	2	1	1	3			
52	70.82	1	2	2	1	2	1			
53	70.98	1	2	2	1	2	2			
54	69.84	1	2	2	1	2	3			
55	75.77	1	2	2	2	1	1			
56	75.77	1	2	2	2	1	2			
57	69.54	1	2	2	2	1	3			
58	75.51	1	2	2	2	2	1			
59	75.51	1	2	2	2	2	2			
60	69.29	1	2	2	2	2	3			
61	69.71	1	2	3	1	1	1			
62	70.10	1	2	3	1	1	2			
63	71.04	1	2	3	1	1	3			
64	69.84	1	2	3	1	2	1			
65	69.84	1	2	3	1	2	2			
66	70.78	1	2	3	1	2	3			
67	69.54	1	2	3	2	1	1			
68	69.54	1	2	3	2	1	2			
69	70.48	1	2	3	2	1	3			
70	69.29	1	2	3	2	2	1			
71	69.29	1	2	3	2	2	2			
72	70.23	1	2	3	2	2	3			
73	71.17	2	1	1	1	1	1			
74	71.46	2	1	1	1	1	2			
75	69.80	2	1	1	1	1	3			
76	72.16	2	1	1	1	2	1			
77	75.77	2	1	1	1	2	2			
78	69.54	2	1	1	1	2	3			
79	75.47	2	1	1	2	1	1			
80	75.47	2	1	1	2	1	2			
81	69.25	2	1	1	2	1	3			
82	75.21	2	1	1	2	2	1			
83	75.21	2	1	1	2	2	2			
84	68.99	2	1	1	2	2	3		8	
85	71.46	2	1	2	1	1	1			
86	76.02	2	1	2	1	1	2			
87	69.80	2	1	2	1	1	3			
88	75.77	2	1	2	1	2	1			
89	75.77	2	1	2	1	2	2			
90	69.54	2	1	2	1	2	3			
91	75.47	2	1	2	2	1	1			
92	75.47	2	1	2	2	1	2			
93	69.25	2	1	2	2	1	3			
94	75.21	2	1	2	2	2	1			
95	75.21	2	1	2	2	2	2			
96	68.99	2	1	2	2	2	3			

	Prob UU Upper & Lower Bounds									
	Tol	Process Curve						Search		
Iter.	Cost	Combinations						1	2	Min
97	69.80	2	1	3	1	1	1			
98	69.80	2	1	3	1	1	2			
99	70.74	2	1	3	1	1	3			
100	69.54	2	1	3	1	2	1			
101	69.54	2	1	3	1	2	2			
102	70.48	2	1	3	1	2	3			
103	69.25	2	1	3	2	1	1			
104	69.25	2	1	3	2	1	2			
105	70.19	2	1	3	2	1	3			
106	68.99	2	1	3	2	2	1			
107	68.99	2	1	3	2	2	2			
108	69.92	2	1	3	2	2	3			
109	72.16	2	2	1	1	1	1			
110	75.77	2	2	1	1	1	2			
111	69.54	2	2	1	1	1	3			
112	75.51	2	2	1	1	2	1			
113	75.51	2	2	1	1	2	2			
114	69.29	2	2	1	1	2	3		5	
115	75.21	2	2	1	2	1	1			
116	75.21	2	2	1	2	1	2			
117	68.99	2	2	1	2	1	3		4	
118	74.96	2	2	1	2	2	1		2	
119	74.96	2	2	1	2	2	2		3	
120	68.74	2	2	1	2	2	3	9	1	min
121	75.77	2	2	2	1	1	1			
122	75.77	2	2	2	1	1	2			
123	69.54	2	2	2	1	1	3			
124	75.51	2	2	2	1	2	1			
125	75.51	2	2	2	1	2	2			
126	69.29	2	2	2	1	2	3			
127	75.21	2	2	2	2	1	1			
128	75.21	2	2	2	2	1	2			
129	68.99	2	2	2	2	1	3			
130	74.96	2	2	2	2	2	1			
131	74.96	2	2	2	2	2	2			
132	68.74	2	2	2	2	2	3		6	min
133	69.54	2	2	3	1	1	1			
134	69.54	2	2	3	1	1	2			
135	70.48	2	2	3	1	1	3			
136	69.29	2	2	3	1	2	1			
137	69.29	2	2	3	1	2	2			
138	70.23	2	2	3	1	2	3			
139	68.99	2	2	3	2	1	1			
140	68.99	2	2	3	2	1	2			
141	69.92	2	2	3	2	1	3			
142	68.74	2	2	3	2	2	1			min
143	68.74	2	2	3	2	2	2			min
144	70.62	2	2	3	2	2	3		7	


Cost Data From UU Unbounded
Assembly Tolerance Cost
72
70
68
66
64
62
60
58
0
12
24
36
48
60
72
84
96
108
120
132
144
Exhaustive Search Iteration


Cost Data From UU Lower Bounded
Assembly Tolerance Cost
72
70
68
66
64
62
60
0
12
24
36
48
60
72
84
96
108
120
132
144
Exhaustive Search Iteration


Cost Data From UU Upper & Lower Bounds
Assembly Tolerance Cost
78
76
74
72
70
68
0
12
24
36
48
60
72
84
96
108
120
132
144
Exhaustive Search Iteration

APPENDIX II

Listings of Sample Data Files and Their Solutions

Problem A

Number of Curves:	10	Assembly Tolerance:	0.014

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.35000000E-02		0				
2	0.0010	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0020	5.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.35000000E-02		0				
2	0.0060	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	8.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.35000000E-02		0				
2	0.0030	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	5.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.35000000E-02		0				
2	0.0010	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0020	5.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	2.0000	0.0000	-1.0000	0.0100	0.00050	0

Solution		
Part	Process	Tolerance
1	1	0.0019600
2	1	0.0061982
3	2	0.0035785
4	1	0.0022633
Asm. Tolerance Cost		21.8650

Problem B

Number of Curves:	13	Assembly Tolerance:	0.023

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.38333333E-02		0				
2	0.0030	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	7.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.38333333E-02		0				
2	0.0020	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	5.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.38333333E-02		0				
2	0.0020	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	4.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.38333333E-02		0				
2	0.0030	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0070	4.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.38333333E-02		0				
2	0.0030	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	3.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.38333333E-02		0				
2	0.0020	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	7.0000	0.0000	-1.0000	0.0100	0.00050	0

Solution		
Part	Process	Tolerance
1	1	0.0046235
2	1	0.0030824
3	3	0.0030824
4	1	0.0040776
5	1	0.0043591
6	1	0.0037751
Asm. Tolerance Cost		29.0499

Problem C

Number of Curves:	15	Assembly Tolerance:	0.040

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.57142857E-02		0				
2	0.0010	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	6.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.57142857E-02		0				
2	0.0020	3.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.57142857E-02		0				
2	0.0030	5.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	4.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.57142857E-02		0				
2	0.0060	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	4.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.57142857E-02		0				
2	0.0010	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.57142857E-02		0				
2	0.0020	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	5.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.57142857E-02		0				
2	0.0030	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0070	8.0000	0.0000	-1.0000	0.0100	0.00050	0

Solution		
Part	Process	Tolerance
1	1	0.0044741
2	1	0.0036531
3	1	0.0057760
4	2	0.0084364
5	1	0.0039458
6	1	0.0059654
7	1	0.0077493
Asm. Tolerance Cost		17.9845

Problem D

Number of Curves:	19	Assembly Tolerance:	0.033

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.41250000E-02		0				
2	0.0010	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	4.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.41250000E-02		0				
2	0.0020	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	6.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.41250000E-02		0				
2	0.0060	5.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	4.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0090	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.41250000E-02		0				
2	0.0030	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	4.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.41250000E-02		0				
2	0.0020	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0070	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.41250000E-02		0				
2	0.0030	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	3.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.41250000E-02		0				
2	0.0050	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0070	7.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.41250000E-02		0				
2	0.0010	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	1.0000	0.0000	-1.0000	0.0100	0.00050	0

Solution		
Part	Process	Tolerance
1	1	0.0026203
2	1	0.0042789
3	3	0.0045385
4	1	0.0045385
5	3	0.0040026
6	2	0.0037057
7	1	0.0071760
8	3	0.0021395
Asm. Tolerance Cost		28.8377

Problem F

Number of Curves: 24 Assembly Tolerance: 0.040

Dimension	Tolerance		Fixed (1)				
CstMod	RefTol	RefCst	FixCst	Power	MaxTol	MinTol	More
1.000000		0.33333333E-02	0				
2	0.0010	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	7.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0030	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	5.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0020	5.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	3.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0060	3.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0070	1.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0020	12.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	6.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0010	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	7.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0060	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0070	7.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0010	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	8.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0030	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	9.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0010	15.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0020	14.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0030	20.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	15.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.33333333E-02	0				
2	0.0010	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	5.0000	0.0000	-1.0000	0.0100	0.00050	0

Solution		
Part	Process	Tolerance
1	1	0.0024385
2	1	0.0035337
3	1	0.0024385
4	2	0.0020402
5	1	0.0037777
6	1	0.0024385
7	2	0.0053978
8	1	0.0023134
9	1	0.0042236
10	1	0.0029865
11	1	0.0059731
12	1	0.0024385
Asm. Tolerance Cost		67.2691

Problem G

Number of Curves:	30	Assembly Tolerance:	0.035

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.50000000E-02		0				
2	0.0020	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	5.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0070	3.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.50000000E-02		0				
2	0.0020	12.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	4.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0100	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.50000000E-02		0				
2	0.0030	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	5.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.50000000E-02		0				
2	0.0010	14.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	12.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	9.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.50000000E-02		0				
2	0.0020	20.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	19.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	17.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0070	12.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.50000000E-02		0				
2	0.0030	4.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	3.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	2.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	1.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.50000000E-02		0				
2	0.0030	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0070	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0090	4.0000	0.0000	-1.0000	0.0100	0.00050	0

Solution		
Part	Process	Tolerance
1	1	0.0048711
2	5	0.0048711
3	1	0.0056597
4	1	0.0040755
5	1	0.0068888
6	4	0.0026680
7	1	0.0059659
Asm. Tolerance Cost		29.5015

Problem H

Number of Curves: 36 Assembly Tolerance: 0.036

Dimension	Tolerance		Fixed (1)				
CstMod	RefTol	RefCst	FixCst	Power	MaxTol	MinTol	More
1.000000	0.30000000E-02		0				
2	0.0010	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0020	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	8.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0010	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	6.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0020	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	4.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0030	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	4.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0010	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	5.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0010	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0020	4.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	3.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0020	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	5.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0040	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	8.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0010	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	6.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0020	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	3.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0030	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	6.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000	0.30000000E-02		0				
2	0.0010	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0020	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	6.0000	0.0000	-1.0000	0.0100	0.00050	0

Solution		
Part	**Process**	**Tolerance**
1	1	0.0025910
2	1	0.0024581
3	1	0.0032774
4	3	0.0025910
5	1	0.0025910
6	1	0.0020070
7	1	0.0030658
8	1	0.0051821
9	1	0.0025910
10	3	0.0031734
11	1	0.0040140
12	1	0.0024581
Asm. Tolerance Cost		53.6233

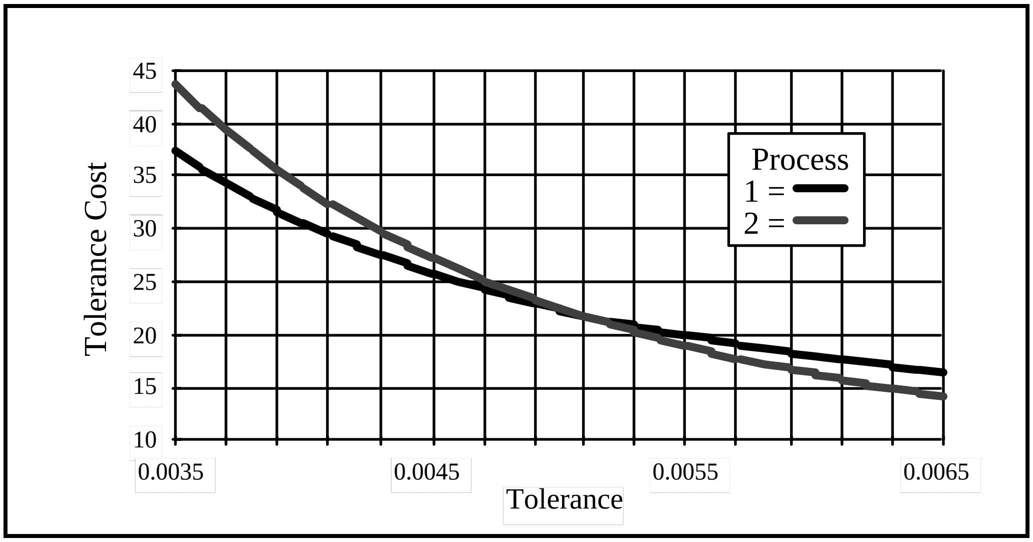
Problem I

Number of Curves: 38 Assembly Tolerance: 0.036

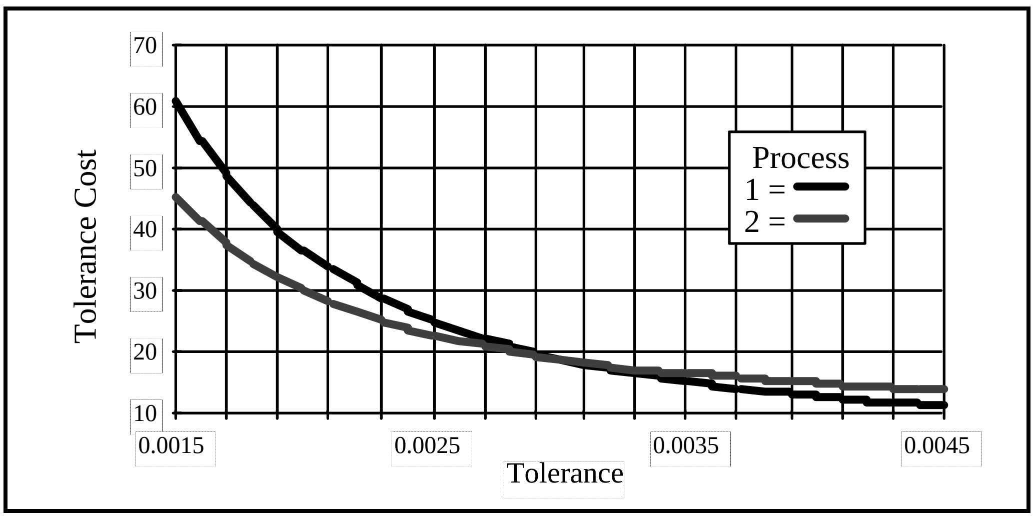
Dimension		Tolerance	Fixed (1)				
CstMod	RefTol	RefCst	FixCst	Power	MaxTol	MinTol	More
1.000000		0.27692308E-02	0				
2	0.0010	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0020	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	8.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0010	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	6.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0020	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	4.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0030	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	4.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0010	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	5.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	2.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0010	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0020	4.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	3.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0020	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0030	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	5.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0040	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	8.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0010	10.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	6.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0020	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	6.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0050	3.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0030	8.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0060	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0080	6.0000	0.0000	-1.0000	0.0100	0.00050	0
1.000000		0.27692308E-02	0				
2	0.0010	9.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0020	7.0000	0.0000	-1.0000	0.0100	0.00050	1
2	0.0040	6.0000	0.0000	-1.0000	0.0100	0.00050	0

```
1.000000     0.27692308E-02   0
 2     0.0010   8.0000   0.0000  -1.0000   0.0100  0.00050      1
 2     0.0020   3.0000   0.0000  -1.0000   0.0100  0.00050      0
```

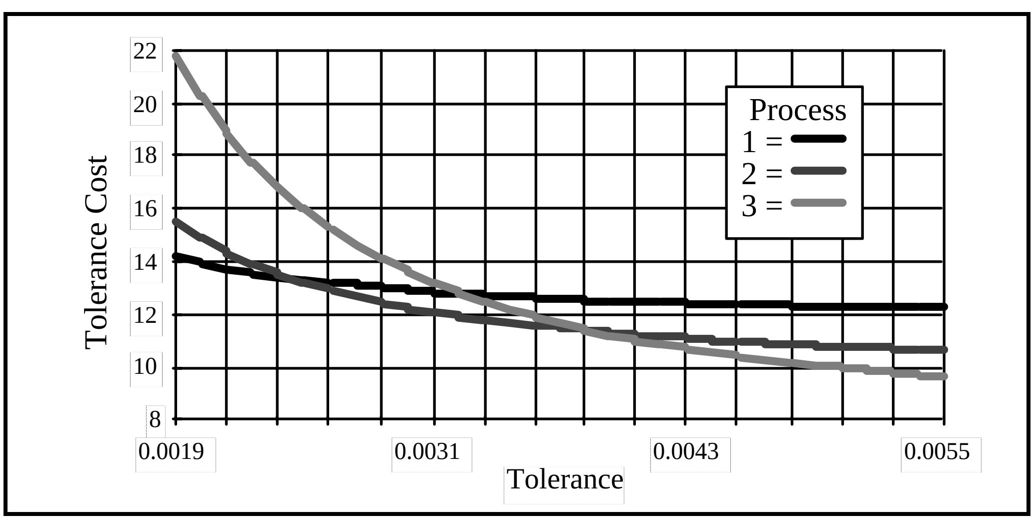
Solution		
Part	Process	Tolerance
1	1	0.0024542
2	1	0.0023283
3	1	0.0031044
4	3	0.0024542
5	1	0.0024542
6	1	0.0019010
7	1	0.0029039
8	1	0.0049084
9	1	0.0024542
10	3	0.0030058
11	1	0.0038021
12	1	0.0023283
13	2	0.0019010
Asm. Tolerance Cost		59.7690



Problem T: Processes for Part 1



Problem T: Processes for Part 2



Problem T: Processes for Part 3

Problem T Upper & Lower Bounds

Number of Curves: 7 Assembly Tolerance: 0.0124

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000		0.20000000E-02	0				
2	0.0060	10.0000	8.0000	-2.0000	0.0080	0.00530	1
2	0.0080	8.0000	2.0000	-2.0000	0.00530	0.0010	0
1.000000		0.30000000E-02	0				
2	0.0030	14.0000	5.0000	-2.0000	0.0030	0.0010	1
2	0.0040	5.0000	10.0000	-2.0000	0.0080	0.0030	0
1.000000		0.20000000E-02	0				
2	0.0010	8.0000	12.0000	-2.0000	0.00250	0.0010	1
2	0.0020	5.0000	10.0000	-2.0000	0.00390	0.00250	1
2	0.0050	2.0000	8.0000	-2.0000	0.0080	0.00390	0

Solution		
Part	Process	Tolerance
1	1	0.0065718
2	2	0.0039806
3	1	0.0018476
Asm. Tolerance Cost		45.7279

Problem T Lower Bounds

Number of Curves: 7 Assembly Tolerance: 0.0124

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.20000000E-02		0				
2	0.0060	10.0000	8.0000	-2.0000	0.01240	0.00530	1
2	0.0080	8.0000	2.0000	-2.0000	0.01240	0.00010	0
1.000000	0.30000000E-02		0				
2	0.0030	14.0000	5.0000	-2.0000	0.01240	0.00010	1
2	0.0040	5.0000	10.0000	-2.0000	0.01240	0.0030	0
1.000000	0.20000000E-02		0				
2	0.0010	8.0000	12.0000	-2.0000	0.01240	0.00010	1
2	0.0020	5.0000	10.0000	-2.0000	0.01240	0.00250	1
2	0.0050	2.0000	8.0000	-2.0000	0.01240	0.00390	0

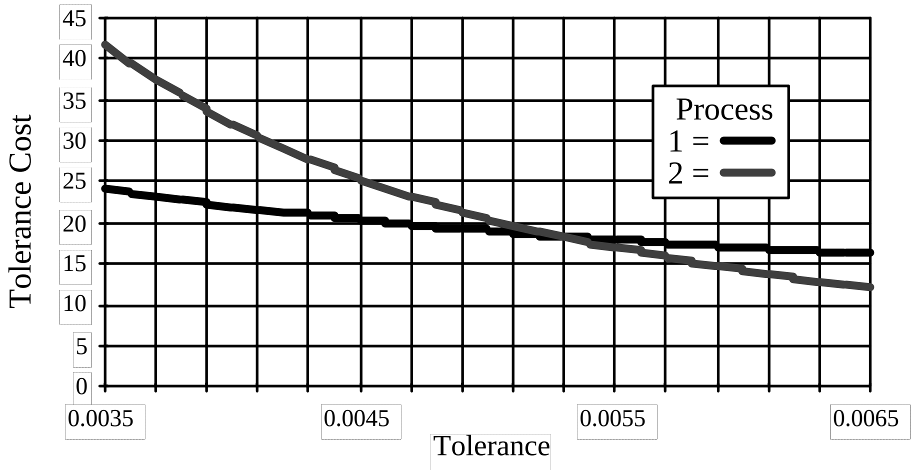
Solution		
Part	Process	Tolerance
1	2	0.0066075
2	1	0.0041407
3	1	0.0016519
Asm. Tolerance Cost		41.0082

Problem T Unbounded

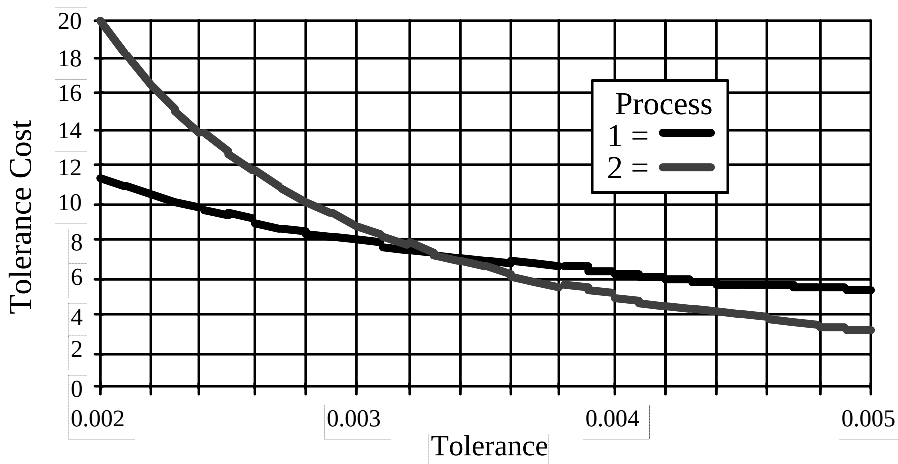
Number of Curves:	7	Assembly Tolerance:	0.0124

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.20000000E-02		0				
2	0.0060	10.0000	8.0000	-2.0000	0.01240	0.00010	1
2	0.0080	8.0000	2.0000	-2.0000	0.01240	0.00010	0
1.000000	0.30000000E-02		0				
2	0.0030	14.0000	5.0000	-2.0000	0.01240	0.00010	1
2	0.0040	5.0000	10.0000	-2.0000	0.01240	0.00010	0
1.000000	0.20000000E-02		0				
2	0.0010	8.0000	12.0000	-2.0000	0.01240	0.00010	1
2	0.0020	5.0000	10.0000	-2.0000	0.01240	0.00010	1
2	0.0050	2.0000	8.0000	-2.0000	0.01240	0.00010	0

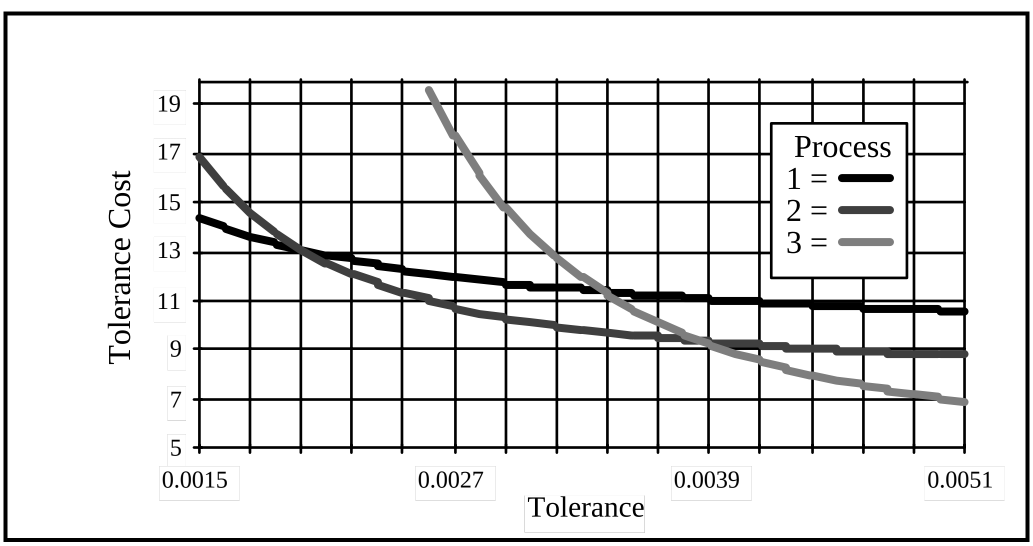
Solution		
Part	Process	Tolerance
1	2	0.0066075
2	1	0.0041407
3	1	0.0016519
Asm. Tolerance Cost		41.0082



Problem U: Processes for Part 1



Problem U: Processes for Part 2



Problem U: Processes for Part 3

Problem U Upper & Lower Bounds

Number of Curves: 7 Assembly Tolerance: 0.0124

Dimension		Tolerance	Fixed (1)				
CstMod	RefTol	RefCst	FixCst	Power	MaxTol	MinTol	More
1.000000		0.20000000E-02	0				
2	0.0060	10.0000	7.0000	-1.0000	0.0080	0.00540	1
2	0.0080	8.0000	0.0000	-2.0000	0.00540	0.0010	0
1.000000		0.30000000E-02	0				
2	0.0030	7.0000	1.0000	-1.0000	0.0080	0.00340	1
2	0.0040	5.0000	0.0000	-2.0000	0.00340	0.0010	0
1.000000		0.20000000E-02	0				
2	0.0010	8.0000	9.0000	-1.0000	0.00350	0.0020	1
2	0.0020	5.0000	8.0000	-2.0000	0.0020	0.0010	1
2	0.0050	2.0000	5.0000	-3.0000	0.0080	0.00350	0

Solution			
Part	Process	Tolerance	
1	2	0.0054000	fix
2	1	0.0034000	fix
3	3	0.0036000	
Asm. Tolerance Cost			35.0931

Problem U Lower Bounds

Number of Curves:	7	Assembly Tolerance:	0.0124

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.20000000E-02		0				
2	0.0060	10.0000	7.0000	-1.0000	0.0080	0.00540	1
2	0.0080	8.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.30000000E-02		0				
2	0.0030	7.0000	1.0000	-1.0000	0.0080	0.00340	1
2	0.0040	5.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.20000000E-02		0				
2	0.0010	8.0000	9.0000	-1.0000	0.0080	0.0020	1
2	0.0020	5.0000	8.0000	-2.0000	0.0080	0.0010	1
2	0.0050	2.0000	5.0000	-3.0000	0.0080	0.00350	0

Solution		
Part	Process	Tolerance
1	2	0.0066031
2	2	0.0035565
3	2	0.0022404
Asm. Tolerance Cost		30.0522

Problem U Unbounded

Number of Curves: 7 Assembly Tolerance: 0.0124

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.20000000E-02		0				
2	0.0060	10.0000	7.0000	-1.0000	0.0080	0.0010	1
2	0.0080	8.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.30000000E-02		0				
2	0.0030	7.0000	1.0000	-1.0000	0.0080	0.0010	1
2	0.0040	5.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.20000000E-02		0				
2	0.0010	8.0000	9.0000	-1.0000	0.0080	0.0010	1
2	0.0020	5.0000	8.0000	-2.0000	0.0080	0.0010	1
2	0.0050	2.0000	5.0000	-3.0000	0.0080	0.0010	0

Solution		
Part	Process	Tolerance
1	2	0.0071950
2	1	0.0027638
3	2	0.0024413
Asm. Tolerance Cost		29.8445

Problem UU Upper & Lower Bounds

Number of Curves: 14 Assembly Tolerance: 0.0248

Dimension		Tolerance	Fixed (1)				
CstMod	RefTol	RefCst	FixCst	Power	MaxTol	MinTol	More
1.000000		0.20000000E-02	0				
2	0.0060	10.0000	7.0000	-1.0000	0.0080	0.00540	1
2	0.0080	8.0000	0.0000	-2.0000	0.00540	0.0010	0
1.000000		0.30000000E-02	0				
2	0.0030	7.0000	1.0000	-1.0000	0.0080	0.00340	1
2	0.0040	5.0000	0.0000	-2.0000	0.00340	0.0010	0
1.000000		0.20000000E-02	0				
2	0.0010	8.0000	9.0000	-1.0000	0.00350	0.0020	1
2	0.0020	5.0000	8.0000	-2.0000	0.0020	0.0010	1
2	0.0050	2.0000	5.0000	-3.0000	0.0080	0.00350	0
1.000000		0.20000000E-02	0				
2	0.0060	10.0000	7.0000	-1.0000	0.0080	0.00540	1
2	0.0080	8.0000	0.0000	-2.0000	0.00540	0.0010	0
1.000000		0.30000000E-02	0				
2	0.0030	7.0000	1.0000	-1.0000	0.0080	0.00340	1
2	0.0040	5.0000	0.0000	-2.0000	0.00340	0.0010	0
1.000000		0.20000000E-02	0				
2	0.0010	8.0000	9.0000	-1.0000	0.00350	0.0020	1
2	0.0020	5.0000	8.0000	-2.0000	0.0020	0.0010	1
2	0.0050	2.0000	5.0000	-3.0000	0.0080	0.00350	0

Solution		
Part	Process	Tolerance
1	2	0.0054000 fix
2	2	0.0034000 fix
3	1	0.0020000 fix
4	2	0.0054000 fix
5	2	0.0034000 fix
6	3	0.0052000
Asm. Tolerance Cost		68.7354

Problem UU Lower Bounds

Number of Curves:	14	Assembly Tolerance:	0.0248

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.20000000E-02		0				
2	0.0060	10.0000	7.0000	-1.0000	0.0080	0.00540	1
2	0.0080	8.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.30000000E-02		0				
2	0.0030	7.0000	1.0000	-1.0000	0.0080	0.00340	1
2	0.0040	5.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.20000000E-02		0				
2	0.0010	8.0000	9.0000	-1.0000	0.0080	0.0020	1
2	0.0020	5.0000	8.0000	-2.0000	0.0080	0.0010	1
2	0.0050	2.0000	5.0000	-3.0000	0.0080	0.00350	0
1.000000	0.20000000E-02		0				
2	0.0060	10.0000	7.0000	-1.0000	0.0080	0.00540	1
2	0.0080	8.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.30000000E-02		0				
2	0.0030	7.0000	1.0000	-1.0000	0.0080	0.00340	1
2	0.0040	5.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.20000000E-02		0				
2	0.0010	8.0000	9.0000	-1.0000	0.0080	0.0020	1
2	0.0020	5.0000	8.0000	-2.0000	0.0080	0.0010	1
2	0.0050	2.0000	5.0000	-3.0000	0.0080	0.00350	0

Solution		
Part	Process	Tolerance
1	2	0.0066031
2	2	0.0035565
3	2	0.0022404
4	2	0.0066031
5	2	0.0035565
6	2	0.0022404
Asm. Tolerance Cost		60.1044

Problem UU Unbounded

Number of Curves: 14 Assembly Tolerance: 0.0248

Dimension CstMod	Tolerance RefTol	RefCst	Fixed (1) FixCst	Power	MaxTol	MinTol	More
1.000000	0.20000000E-02	0					
2	0.0060	10.0000	7.0000	-1.0000	0.0080	0.0010	1
2	0.0080	8.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.30000000E-02	0					
2	0.0030	7.0000	1.0000	-1.0000	0.0080	0.0010	1
2	0.0040	5.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.20000000E-02	0					
2	0.0010	8.0000	9.0000	-1.0000	0.0080	0.0010	1
2	0.0020	5.0000	8.0000	-2.0000	0.0080	0.0010	1
2	0.0050	2.0000	5.0000	-3.0000	0.0080	0.0010	0
1.000000	0.20000000E-02	0					
2	0.0060	10.0000	7.0000	-1.0000	0.0080	0.0010	1
2	0.0080	8.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.30000000E-02	0					
2	0.0030	7.0000	1.0000	-1.0000	0.0080	0.0010	1
2	0.0040	5.0000	0.0000	-2.0000	0.0080	0.0010	0
1.000000	0.20000000E-02	0					
2	0.0010	8.0000	9.0000	-1.0000	0.0080	0.0010	1
2	0.0020	5.0000	8.0000	-2.0000	0.0080	0.0010	1
2	0.0050	2.0000	5.0000	-3.0000	0.0080	0.0010	0

Solution		
Part	Process	Tolerance
1	2	0.0069015
2	1	0.0025964
3	2	0.0023417
4	2	0.0069015
5	2	0.0037172
6	2	0.0023417
Asm. Tolerance Cost		59.6711