A Survey of Research in the Application of Tolerance Analysis to the Design of Mechanical Assemblies

ADCATS Report No. 91-1

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Published in Research in Engineering Design (1991) 3:23-37

April 5, 1991

Abstract

Tolerance analysis is receiving renewed emphasis as industry recognizes that tolerance management is a key element in their programs for improving quality, reducing overall costs and retaining market share. The specification of tolerances is being elevated from a menial task to a legitimate engineering design function. New engineering models and sophisticated analysis tools are being developed to assist design engineers in specifying tolerances on the basis of performance requirements and manufacturing considerations.

This paper presents an overview of tolerance analysis applications to design with emphasis on recent research that is advancing the state of the art. Major topics covered are:

1) New models for tolerance accumulation in mechanical assemblies, including the Motorola Six Sigma model.
2) Algorithms for allocating the specified assembly tolerance among the components of an assembly.
3) The development of 2-D and 3-D tolerance analysis models.
4) Methods which account for non-Normal statistical distributions and nonlinear effects.
5) Several strategies for improving designs through the application of modern analytical tools.

1 Introduction

Interest in tolerance analysis is rapidly increasing in industry. The quest for quality has focused attention on the effects of variation on cost and performance of manufactured products. Excess cost or poor performance will eventually show up as a loss of market share. Therefore, the specification of tolerance limits on each dimension and feature of engineering drawings is considered by many to be a vital design function. Tolerance requirements have a far-reaching influence that touches nearly every aspect of the manufacturing enterprise as shown in Fig. 1.

Both engineering design and manufacturing personnel are concerned about the effects of tolerances. Engineers like tight tolerances to assure fit and function of their designs. Manufacturers prefer loose tolerances which make parts easier and less expensive to produce. Therefore tolerance specifications become a critical link between engineering and manufacturing, a common meeting ground where competing requirements may be resolved.

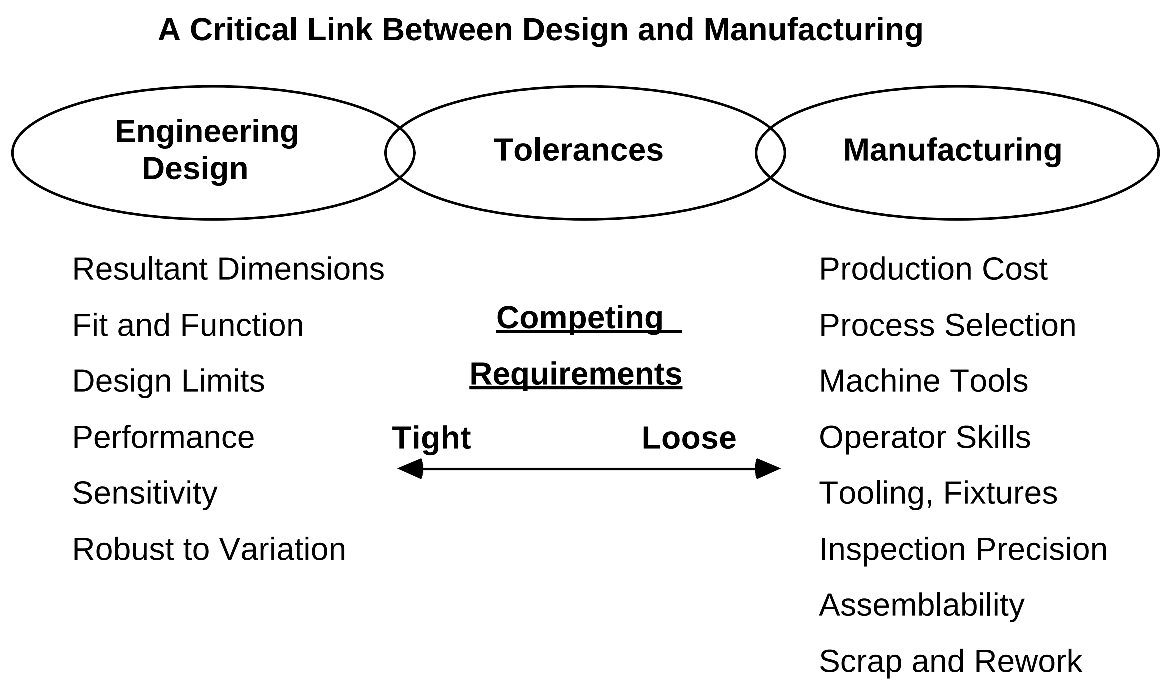


Fig. 1. The effects of assigned tolerances are far-reaching.

In the last few years, numerous companies have established comprehensive programs in quality management. Noteworthy among them are the efforts of Motorola, IBM and Xerox,who have initiated formal, corporate-wide programs for improved tolerance specification, monitoring and control. Their success in reducing waste while cutting cost and development time and reclaiming lost market share has received national praise [Placek 1989a, Placek 1990, Kendrick 1991].

Another indication of the growing interest in tolerancing is the Mechanical Tolerancing Workshop sponsored by NSF and ASME in 1988 which brought together an international group of experts in tolerancing to discuss the state of the art and identify research opportunities [Paleck, 1989b, ASME 1990]. This has been followed by special theme sessions at several ASME conferences, such as the Design Technical Conference in Montreal (1989), the Design Show in Chicago (1990), and the Computers in Engineering Conference in Boston (1990).

2 Models for Tolerance Accumulation

The basis for rational tolerance specification is to create an analytical model to predict the accumulation of tolerances in a mechanical assembly. Critical clearances or fits or other resultant features of an assembly are generally controlled by the stackup or sum of several component tolerances. A number of analytical models exist with varying levels of sophistication as shown in Fig. 2.

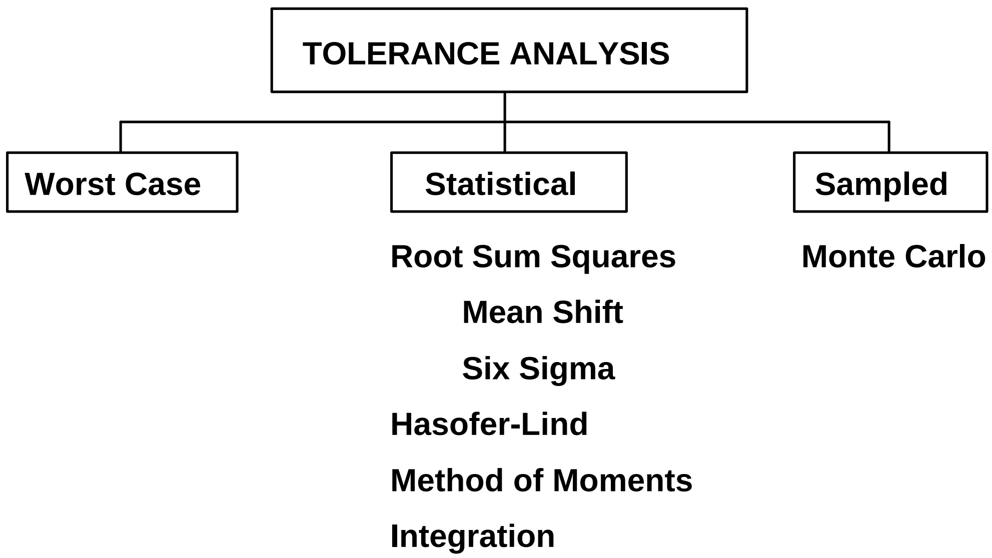


Fig. 2. Mathematical models of tolerance accumulation.

Common models for predicting how component tolerances sum are Worst Case (WC) and Root Sum Square (RSS) as shown in Eqs. 1 and 2 [Fortini 1967].

One-dimensional assemblies: **Two- or three-dimensional assemblies:**

Worst Case

$$dU = \Sigma\, T_i \le T_{ASM} \qquad dU = \Sigma\left(\left|\frac{\partial f}{\partial x_i}\right| T_i\right) \le T_{ASM} \qquad (1)$$

Root Sum Square

$$dU = \left[\Sigma\, T_i^2\right]^{1/2} \le T_{ASM} \qquad dU = \left[\Sigma\left(\frac{\partial f}{\partial x_i}\right)^2 T_i^2\right]^{1/2} \le T_{ASM}$$

(2)

where X_i are the nominal component dimensions, T_i are the component tolerances, dU is the predicted assembly variation, T_{ASM} is the specified limit for dU, and $f(X_i)$ is the assembly function describing the resulting dimension of the assembly, such as the clearance or interference.

The partial derivatives $\partial f/x_i$ represent the sensitivity of the assembly tolerance to variations in individual component dimensions. For a one-dimensional tolerance stack the sensitivity is ±1.0.

The Worst Case model assumes all the component dimensions occur at their worst limit simultaneously. It is used by designers to assure that all assemblies will meet the specified assembly limit. However, as the number of parts in the assembly sum increases, the component tolerances must be greatly reduced in order to meet the assembly limit, requiring higher production costs. In the RSS model, the low probability of the worst case combination occurring is taken into account statistically, assuming a Normal or Gaussian distribution for component variations. Tolerances are commonly assumed to correspond to six standard deviations (6σ or ±3σ). Component tolerances may be increased significantly since they add as the root sum squared (RSS).

Statistical distributions may be used to predict the yield of an assembly, that is, the number or fraction of assemblies which are likely to lie inside the spec limits. RSS analysis generally predicts too few rejects when compared to real assembly processes. This is due to the fact that the Normal distribution is only an approximation of the true distribution, which may be flatter or skewed. The mean of the distribution may also be shifted from the midpoint of the tolerance range. To account for these uncertainties, a more general form of the RSS model is frequently used:

$$dU = C_f Z \left[\sum \left(\frac{\partial f}{\partial x_i}\right)^2 \left(\frac{T_i}{Z_i}\right)^2 \right]^{1/2} \leq T_{ASM} \qquad (3)$$

where Z is the number of standard deviations desired for the specified assembly tolerance and Z_i describes the expected standard deviations for each component tolerance. C_f is a correction factor added to account for any non-ideal conditions. Typical values for C_f range from 1.4 to 1.8 [Bender 1968, Levy 1974, Gladman 1980].

Another conservative estimate of assembly tolerances assumes the component dimensions are uniformly distributed over the specified tolerance range. In this case, the value of Z_i is 3. If truncated normal distributions arise due to inspection of component parts, then choose $3 < Z_i < 3$, as described by Spotts [1983]. Spotts also suggested calculating the Worst Case and RSS assembly tolerance and simply averaging the two as a safe estimate [Spotts 1978].

2.1 Estimated Mean Shift Model

Simple RSS analysis assumes that the variation of each component dimension is symmetrically distributed about the mean or nominal dimension. However, in real processes, the mean is shifted due to setup errors or drifts due to time-varying parameters such as tool wear. Ignoring mean shifts can be very detrimental, resulting in large errors in estimates of the number of assemblies within spec limits [Spotts 1978, Evans 1975b].

Further modifications to the RSS model have been proposed to take into account mean shifts or biased distributions. Mansoor [1963] proposed that the tolerance accumulation be represented as a WC sum plus a RSS sum. A similar model by Greenwood [1987], Greenwood and Chase [1987] and Chase and Greenwood [1988] introduced an estimated mean shift factor m_i (a number between 0 and 1.0) which quantifies the expected mean shift as a fraction of the specified tolerances. Eq. 4 illustrates the resulting expression.

Worst Case Sum Statistical Sum

$$dU = \Sigma \subseteq m_i \frac{\partial f}{\partial x_i} T_i \subseteq + \left[\Sigma \left((l - m_i)^2 \frac{\partial f}{\partial x_i}^2 T_{i^2} \right) \right]^{1/2} \leq T_{ASM} \qquad (4)$$

This is a versatile model. If all the m_i are set to 1.0, the result is a WC model. If all the m_i are set to 0, the result is a RSS model. By selecting m_i between 0. and 1.0, the resulting variation will always lie between the WC and RSS predictions. And any combination of m_i may be chosen to account for the degree of uncertainty in individual process characterizations.

2.2 Motorola Six Sigma Program

A new Tolerance accumulation model that has caught the attention of industry was developed by the Motorola Corp. as a basic element of their award-winning Six Sigma quality program which is now being adopted by other leading companies[Placek 1989a].

The basic premise of the Six Sigma Program is: in order to achieve high quality in a complex product comprised of many components and processes, each component and process must be produced at significantly higher quality levels in order for the composite result to meet final quality standards. Stated statistically, suppose there were 1000 dimensions or other characteristics of your product, any one of which could lower the quality of the finished product. If each characteristic were produced to $\pm 3\sigma$ quality (99.73% acceptable parts or 2700 defects per million), the resultant assemblies would be only $(.9973)^{1000}$=.067 or 6.7% defect free. To have 99.73% defect free assemblies, you would need to produce each component to a quality of $(.9973)^{.001}$=.9999973 or 99.99973%, which is 2.7 defects per million.

To achieve the high quality levels required for world competition in the electronics industry, Motorola has mandated $\pm 6\sigma$ quality for all processes (.002 defects per million). However, they also recognize that shifts and drifts in the mean of the processes are expected, so they have introduced a modified process model which includes an allowance for accumulated mean shifts. The result is a net quality level of $\pm 4.5\sigma$ (3.4 defects per million).

A versatile feature of the Six Sigma model of a process is the ability to distinguish between short term and long term process capability. The process capability quantifies the spread of the process. It is defined as 6.0 times the standard deviation of the process, $6\sigma_i$. Over the long term, however, the mean of a process will drift due to tool wear or the set up will vary from lot-to-lot, resulting in an apparent increase in the process capability. The resulting modified standard deviation of a component process may be estimated from Eq. 5

$$\sigma_i = \frac{T_i}{3C_{pi}(1-m_i)} \qquad \text{where} \quad C_{pi} = \frac{UL - LL}{6\sigma_i} \qquad (5)$$

UL and LL are the upper and lower tolerance limits, respectively, C_{pi} is the Process Capability Ratio, or the ratio of the specified tolerance range to the process capability. Variable m_i is the Mean Shift Factor. When $m_i = 0$, σ_i describes the short term variation in the process. When

$0 < m_i < 1.0$, σ_i approximates the long-term variation. For the standard Six Sigma model, the target values of $C_{pi} = 2$ and $m_i = 0.25$ result in tolerance limits of $\pm 4.5\sigma_i$ over the long term. Note that other C_{pi} and m_i values may be selected to account for the degree of uncertainty in individual process characterizations.

The Six Sigma model for tolerance accumulation is shown in Eq. 6. It accounts for process mean shift variations by using an effective standard deviation as expressed in Eq. 5. If a value for σ_i for the process is known from prior experience, it may be substituted in the equation.

$$dU = Z\ \left[\Sigma\left(\frac{\partial f}{\partial x_i}\right)^2\left(\frac{T_i}{3C_{p_i}(1-m_i)}\right)\right]^{1/2} \leq T_{ASM} \qquad (6)$$

Of course, tolerance analysis of assemblies is only one component of the complete Motorola quality management system, but the Six Sigma tolerance analysis model is a significant contribution. It is more realistic and versatile than the models commonly used for design. It should have a major impact on reducing production costs and improving quality [Harry et al. 1987, 1988, 1990].

3 2-D Tolerance Analysis

Modeling tolerance accumulation in 2-D assemblies is much more difficult than for 1-D. Component dimensions are joined together as 2-D vector chains or loops. The loops pass from mating part-to-mating part, passing through the points of contact between parts. Vector loops may be open or closed. Each open loop describes a resultant assembly clearance or interference. Closed loops describe assemblies containing some adjustable element, such as a spring-loaded part or fastener, which takes up the slack and assures closure.

Complex assemblies may require several open and closed loops to define assembly relationships. Each vector loop results in three scalar equations which describe the response to manufacturing variations. The resulting nonlinear system of equations must be solved simultaneously for the assembly resultants [Fortini 1967, Chase & Greenwood 1988, Marler 1988, Bjorke 1989].

3.1 Kinematic Adjustments

Manufacturing variations propagate through an assembly by small kinematic adjustments between mating parts. Thus, kinematic constraints must be applied to the vector loops. Each kinematic constraint introduces kinematic degrees of freedom into the model, such as the location of a point of contact on a sliding plane or the relative angle between two mating parts which are joined by a pin joint. The nominal values of the kinematic variables are not known. They are not dimensioned on any engineering drawing. They must be determined by assembling an ideal assembly for which all manufactured dimensions are at their nominal values. They may then be calculated by solving the set of vector loop equations and kinematic constraints describing the system.

The variations in the kinematic variables are also unknown. Small changes in the manufactured dimensions produce small changes in the kinematic variables. Thus, the kinematic variables are dependent variables. The variation in the kinematic variables may be determined by specifying the manufacturing variations (design tolerances) and solving the vector loop equations for the resulting adjustments in the assembly.

Thus, there are two steps in solving the vector equations. First, set all the manufactured dimensions to their nominal values and solve the system of vector loop equations for the nominal values of the kinematic variables and assembly resultants. Vector equations are generally nonlinear and must be solved by iterative means. If the nominal values may be determined from, say, a precise CAD layout, this step may be omitted.

Second, linearize the equations for small variations about the nominal by Taylor's series expansion, retaining first order derivatives. Substitute the design tolerances and solve this system of linearized equations for the corresponding variation in the kinematic variables and assembly resultants. [Marler 1988, Chase et al. 1989]

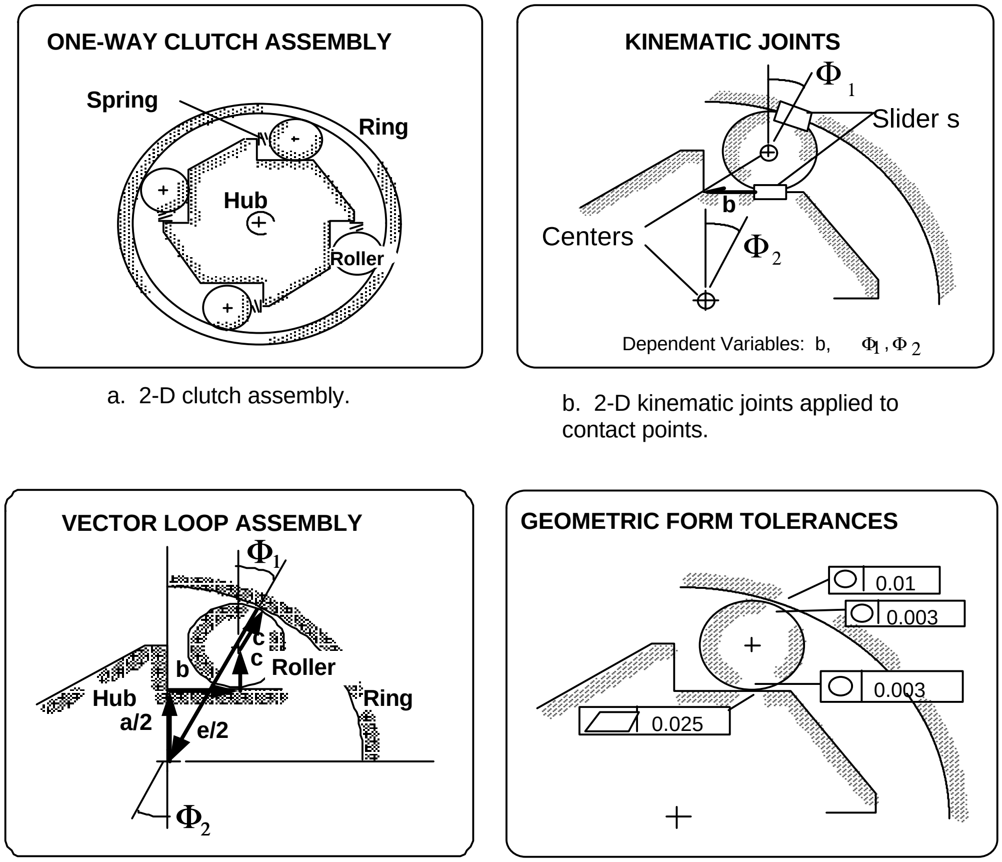
3.2 Geometric Tolerances

Geometric tolerances are being used increasingly by aerospace and other industries to assure form and function of mechanical parts. They are distinct from size or dimensional tolerances. They control the form and orientation (flatness, roundness, perpendicularity, etc.) and location (position, concentricity, etc.) of surfaces and other features as defined in the standard, ANSI Y14.5 [ASME 1983]. Recommended practice is to establish geometric tolerances on the basis of "Maximum Material Condition" (MMC), which is essentially a Worst Case analysis, resulting in tight component tolerances. The standard does not include statistical considerations or tolerance accumulation effects [Foster 1986, Levy 1974].

However, geometric deviations provide sources of variation that can accumulate and propagate through an assembly the same as size tolerances. Depending on the number of components and the geometry, they can have a significant influence on the resultant assembly variations. Efforts are being made to analyze geometric tolerances statistically and include geometric variations in vector loop models in order to compute their effects on complex assemblies along with dimensional variations [Chun 1988, Chase et al. 1989].

3.3 Sample 2-D Problem

Fig. 3a illustrates a simple 2-D problem described by Fortini [1967]. It is a drawing of a one-way mechanical clutch.



a. 2-D clutch assembly.

b. 2-D kinematic joints applied to contact points.

c. Vector loop model of clutch.

d. Addition of geometric form tolerances.

Fig. 3. Vector loop model of a clutch assembly.

This is a common device used to transmit rotary motion in only one direction. When the outer ring of the clutch is rotated clockwise, the rollers wedge between the ring and hub, locking the two so they rotate together. In the reverse direction, the rollers just slip, so the hub does not turn. The angle $\mathbf{\Phi_1}$ between the two contact points is critical to the proper operation of the clutch. If $\mathbf{\Phi_1}$ is too large, the clutch will not lock; if it is too small the clutch will not unlock.

The primary objective of performing a tolerance analysis on the clutch is to determine how much the angle $\mathbf{\Phi_1}$ is expected to vary due to manufacturing variations in the clutch components. The independent manufacturing variables are the hub dimension **a**, the cylinder radius **c**, and the ring diameter **e**. The dependent assembly resultants are the location of contact point, **b**, and the two angles $\mathbf{\Phi_1}$ and $\mathbf{\Phi_2}$.

In this example, the designer begins the construction of a vector kinematic model by creating a precise CAD model of the assembly, to which the vector model will be added as an overlay. On the CAD model, he defines the kinematic joints at the contact locations between parts (Fig. 3b). When the parts are properly constrained by the joints, they will have zero kinematic degrees of freedom.

Next, the joints are connected by vectors forming a closed vector loop and making sure all critical dimensions have been included in at least one loop (Fig. 3c). The geometry from the loops and joints is used to calculate the sensitivity of the dependent kinematic dimensions to changes in each independent dimension. This information is used for computation of the expected variation in assembly resultants and possible re-allocation of tolerances.

Geometric form tolerances may be added at each contact joint (Fig. 3d). The sensitivity of the dependent assembly dimensions **b**, $\mathbf{\Phi_1}$ and $\mathbf{\Phi_2}$ to form and orientation variations may then be calculated. The predicted variation in assembly resultants may then be estimated by modifying Eq. 2 to include form variations:

$$dU = \left[\Sigma \left(\frac{\partial f}{\partial x_i} \right)^2 T_i^2 + \Sigma \left(\frac{\partial f}{\partial a_i} \right)^2 d\alpha_i^2 \right]^{1/2} \leq T_{ASM} \qquad (7)$$

where $d\alpha_i$ are the form variations and dU is the resultant assembly variation $\mathbf{d\Phi_1}$.

Once an expression for $\mathbf{d\Phi_1}$ has been obtained, tolerance design may proceed. By substituting reasonable values for the component tolerances T_i and form tolerances $d\alpha_i$ into Eq. 7, the designer must verify that the predicted variation in $\mathbf{\Phi_1}$ will be less than design requirement T_{ASM}. Alternately, to reduce production cost, one might set $\mathbf{d\Phi_1}$ equal to T_{ASM} and solve for the largest possible values for T_i.

4 3-D Tolerance Analysis

4.1 3-D Solid Models

An alternative approach to vector assembly models is to create a solid model of the assembly on a CAD system. The solid model serves as the assembly function. Small changes can be simulated and their effects will propagate realistically, provided each part is located relative to its adjacent parts and provided that kinematic adjustments are permitted.

Solid models are precise representations of assembly geometry. They are constructed from surfaces or solid primitives. The main obstacle to the use of commercial solid modelers for tolerance analysis is the lack of conventional dimension and tolerance data. This imposes a great hardship upon software application developers. The trend toward feature-based solid modelers may help to overcome this deficiency.

To obtain the sensitivities required for calculating an assembly tolerance sum, the relationship between the parameters defining the model surfaces or solid primitives and the dimensioned quantities appearing on an engineering drawing of the parts must be determined. This may be accomplished by making small changes in each of the model variables, measuring the resultant change in the component dimensions and assembly resultants as shown in Fig. 4 and computing the corresponding sensitivities. The sensitivities are used to form the linearized expressions, Eq. (8), relating the variations in the component dimensions and assembly resultants to variations in the model parameters. Finally, a linear programming problem is set up to find a set of model variations, dM_j, which, when substituted into Eq. (8), satisfies the specified component and assembly tolerance limits [Turner and Wozney 1987, 1990, Turner ,Wozney and Hoh 1987].

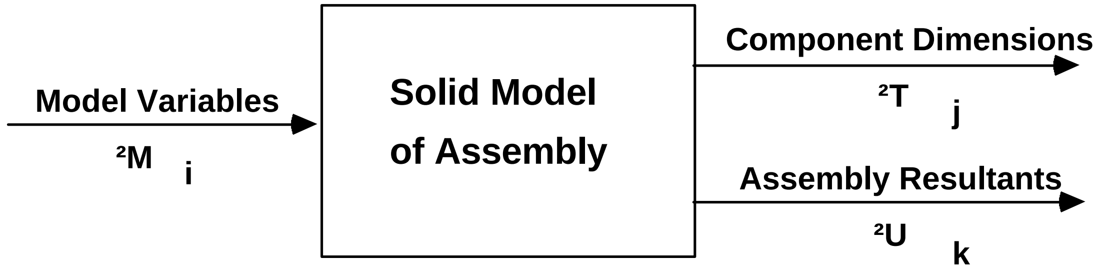


Fig. 4. Computing tolerance sensitivities from the solid model of an assembly.

$$\text{Component tolerances:} \qquad dX_i = \sum \frac{\partial f}{\partial M_j}\, dM_j \le T_i$$

$$\text{Assembly tolerances:} \qquad dU_i = \left[\Sigma\left(\frac{\partial g}{\partial M_j}\right)^2 dM_j^2\right]^{1/2} \le T_{ASM} \qquad (8)$$

Solid modelers are CPU intensive. Changing a single parameter for a sensitivity calculation requires regeneration of the entire CAD geometry. A detailed model of an assembly may have thousands of model parameters, resulting in a substantial wait for a complete sensitivity calculation on all but the most powerful computers. However, significant progress is being made in reducing the enormous number of sensitivity calculations by prior examination of the model to eliminate non-contributing parameters [Martino and Gabriele 1989a, 1989b]. Current research efforts by Turner include the addition of kinematic constraints [Turner 1990, Turner and Srikanth 1990, Srikanth and Turner 1990].

A number of researchers are taking an axiomatic approach to 3-D tolerance representations in solid models. Requicha represents the model variations as a pair of "offset boundaries," or offset surfaces, which bound each ideal surface. The set of offset boundaries form a tolerance zone which bounds the entire part [Requicha 1983,1986]. A similar definition creates a "virtual boundary" formed by taking into account the combined effects of all applicable size and form tolerances. [Jayaraman and Srinivasan 1989, Srinivasan and Jayaraman 1989]. Several problems remain to be resolved, including: potential conflicts with existing standards, incorporation of statistical models and the lack of kinematic assembly interactions. For commentaries on these issues, the reader is referred to [Faux 1986, Etesami 1987].

Variational geometry is another fundamental approach. It requires the formulation of analytical equations describing the geometric relationships which must be maintained in an assembly. Constraints such as perpendicular surfaces or surfaces in sliding contact are defined in terms of dimensional parameters. If the design is modified, the system of equations may be solved to adjust the free variables in keeping with the constraints. The advantages of this method are the ease of design iteration and the realistic propagation of manufacturing variations by kinematic adjustments. However, the resulting system of nonlinear equations can become very large and must be solved simultaneously. Also, geometric form and feature tolerances must still be taken into account [Light and Gossard 1982, Gossard et al. 1988, Chung and Schussel 1990].

4.2 3-D Vector Models

Vector loop models of assemblies may be applied to 3-D assemblies. The vectors are not confined to a plane and the kinematic conditions become more complex. The system of equations is twice as large, since each 3-D vector equation yields six scalar equations. Form and orientation tolerances may be added. Their interaction with the kinematic joint axes must be carefully modeled. Recently, vector models of an assemblies have been overlaid on the corresponding 3-D solid model and associated with that model such that changes in the solid model are automatically reflected in the vector model [Robison 1989].

One major advantage of 3-D vector models of assemblies over solid models is that the geometry is reduced to only those parameters required to perform a tolerance analysis. Sensitivity analysis is much simpler. It is very efficient computationally and well suited to design iteration. Describing manufacturing variations with vectors and kinematics is also a medium that engineering designers are already familiar with.

5 Tolerance Analysis of Mechanisms

Since tolerance analysis of assemblies involves the creation of a kinematic model, the principal differences from the classical kinematic analysis of mechanisms are the inputs and the magnitude of the motions. A mechanism will have one or more kinematic variables as a prescribed input. The nonlinear system of kinematic equations will be solved for the remaining kinematic variables. The resulting motions will be orders of magnitude larger than those due to manufacturing variations.

For tolerance analysis of a mechanism, the dimensions of the various components will be held fixed while the nominal position and orientation of each component is determined by kinematic analysis. Then a tolerance analysis may be performed by introducing manufacturing variations while holding the mechanism stationary. This may be repeated at selected positions considered of interest or the mechanism may be incremented at regular intervals to see the results of manufacturing variations over a range of motion.

References to mechanism tolerance analysis:

Mechanism	**Analysis**	**Authors**
4-bar function generator	tolerance estimating	[Garrett & Hall 1969]
	tolerance allocation	[Dhande & Chakraborty 1973]
	tolerance allocation	[Agarwal 1981]
4-bar path generator	tolerance estimating	[Baumgarton & Van der Werff 1985]
	adjustable linkages	[Schade 1982]
	tolerance allocation	[Mallik & Dhande 1987]
Slider-crank	tolerance allocation	[Fu et al 1987, Schade 1980]
General 2-D linkages	tolerance allocation	[Fenton et al 1989]

3-D 4-bar function generator	tolerance allocation	[Dhande & Chakraborty 1978]
	tolerance allocation	[Beohar & Rao 1980]
Disk cams	tolerance allocation	[Rao & Gavane 1980]

6 Nonlinear Analysis, Non-Normal Distributions

6.1 Nonlinear Analysis

The linearized models for tolerance accumulation in an assembly, as expressed in Eqs. 1 through 4, assume that the sensitivity, evaluated at the nominal, is constant over the tolerance limits. That is, if you evaluated the assembly function as you varied one parameter over its tolerance range, the slope of the function would be nearly constant . This is usually a reasonable assumption, when the tolerances are very small compared to the nominal dimensions or when there are a large enough number of components to mask the effects.

In a highly nonlinear assembly, the sensitivity may not be symmetric over this range and the distribution of the assembly resultant will be skewed or asymmetric. This can happen even though all of the component distributions are symmetric. A linearized model, however, will always yield a symmetric resultant from symmetric inputs.

Analysis methods which can treat nonlinear effects are shown in Table 1. While Hasofer-Lind retains nonlinear effects, it is limited to Normal distributions. The relative CPU efficiency values are only estimates, based on the author's own experience. Actual values could differ substantially depending on problem complexity.

Table 1. Comparison of Tolerance Analysis Methods

	Assembly Model		Distributions		Efficiency
Analysis Method	**Linearized**	**Nonlinear**	**Normal**	**Non-Normal**	**Relative CPU Time**
Worst Case	X	X	NA	NA	1
RSS	X		X		1
Hasofer-Lind		X	X		6
Method of Moments		X	X	X	10
Integration		X	X	X	60
Monte Carlo	X	X	X	X	100

6.2 Non-Normal Distributions

Designers seldom have sufficient data by which to specify the distribution of the manufacturing processes. Data has not been gathered because the parts have not yet been made. Tooling has not been ordered nor certified. The processes may not have been selected. It is customary in such cases for designers to assume a Normal distribution. If there is uncertainty about the process, then a Uniform distribution may be assumed. Generally, deviations from Normal are slight and vary from batch to batch, making it difficult to predict. If the component distributions have a strong central tendency, and there are five or more components contributing to an assembly sum, the result is likely to approximate a Normal distribution regardless of the component distributions. If there are mean shifts, one of the models discussed earlier may be used.

However, in some cases, where the process has been well characterized and is known to exhibit skewness (asymmetry) or kurtosis (peakedness), it may be justifiable to estimate these parameters and apply an advanced analysis method, such as Monte Carlo or Method of Moments. This is more likely to occur after production has begun and data has been gathered on finished parts.

6.3 Monte Carlo Simulation

Monte Carlo Simulation is a powerful tool for tolerance analysis of mechanical assemblies, for both nonlinear assembly functions and non-Normal distributions. It is based on the use of a random number generator to simulate the effects of manufacturing variations on assemblies. Fig. 5 illustrates the method.

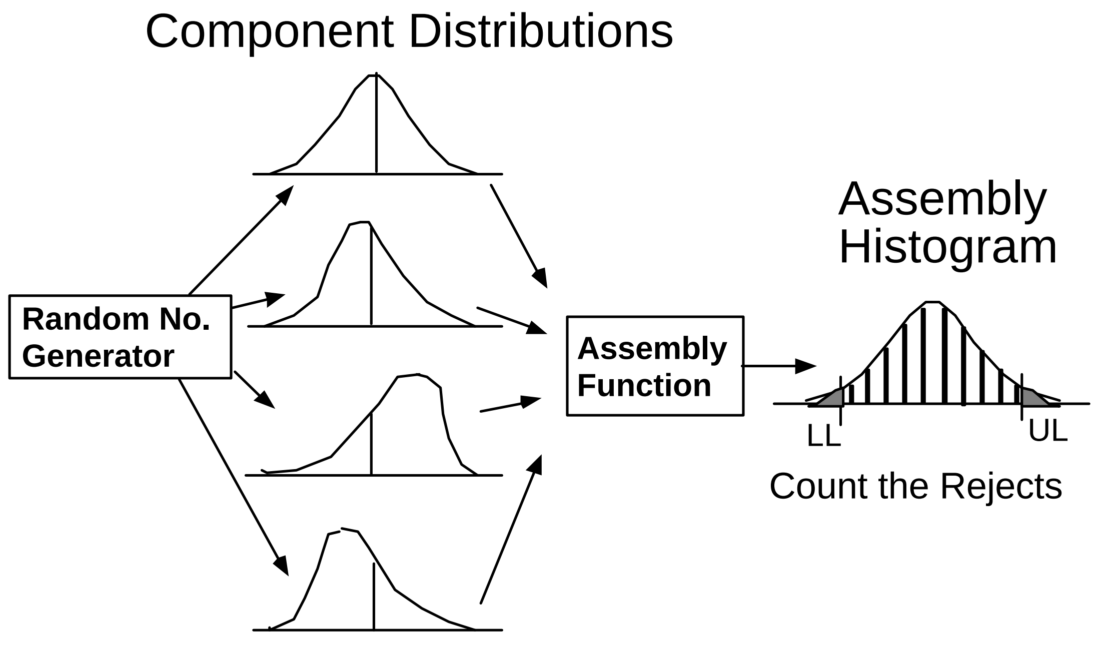


Fig. 5. Assembly tolerance analysis by Monte Carlo simulation.

The Monte Carlo method consists of the following steps:

1. A critical assembly resultant is identified and design limits are specified.
2. The component dimensions which contribute to the critical resultant are identified and tolerances are specified for each dimension.
3. Also specified is the statistical distribution for the variation in each component dimension. The distribution may be described algebraically or empirically.
4. An assembly function is formulated relating the component dimensions to the resultant assembly dimension.

5. A set of component dimensions for a single assembly is selected using a random number generator to apply a small variation to each dimension. The resultant assembly dimension is calculated by means of the assembly function and compared to the assembly limits to determine if it is within spec.

6. Step 5 is repeated until a sufficient number of assemblies has been simulated to plot a histogram and estimate the percent of assemblies that would be rejected based on the specified tolerances. A variation on this step is to fit a distribution to the histogram and use the distribution function to calculate the percent rejects.

The biggest disadvantage of the Monte Carlo method is that it requires large samples to achieve reasonable accuracy. The number of simulated assemblies must be on the order of 100,000 to 400,000 to predict the small percentage rejects of modern manufacturing processes. For example, for a $\pm 3\sigma$ assembly spec, a sample of 100,000 assemblies should yield 135 rejects at each limit, but could be 10 to 20% off [Shapiro and Gross 1981]. Design iterations get pretty tedious when 100,000 simulations must be generated for each trial design. The designer is not likely to have the patience to search for the optimum design.

References to nonlinear or non-Normal analysis include:

Method	**Authors**
Worst Case	Greenwood & Chase [1988a]
Hasofer-Lind	Parkinson [1982, 1985], Lee&Woo [1990], Greenwood & Chase[1988b]
Method of Moments	Evans [1970, 1974, 1975a, 1975b], Cox [1979, 1986], Shapiro & Gross [1981], Greenwood [1987]
Integration	Evans [1967, 1971, 1972], Sorensen [1990]
Monte Carlo	Grossman [1976], Shapiro & Gross [1981], DeDoncker & Spencer [1987], Craig [1989], Doepker & Nies [1989], Early & Thompson [1989]

7 Design Improvement

Design improvement is the principal aim of tolerance analysis. Rather than just predict the effects of variations, the goal is to systematically select tolerances throughout an assembly to assure that design requirements will be met. The ideal assigned tolerances not only assure acceptable performance, but also assure that parts can readily be produced and assembled, resulting in high process yields and reduced costs. Several strategies for design improvement exist. A good general discussion is presented in the excellent book by Spence and Soin [1988]. Although the book is applied to electronics design, the methods are just as applicable to mechanical systems.

The main topics in design improvement are: Yield Modification, Tolerance Allocation, Sensitivity Analysis, and Probabalistic Design and Design Optimization.

7.1 Yield Modification

The yield of an assembly process may be increased by design centering. The procedure is illustrated in Fig. 6a. The figure shows the effect of varying two design parameters, p_1 and p_2. The region defined by shaded boundaries is called the feasible design space and represents the limits set on the values of p_1 and p_2 for acceptable performance. The rectangle represents the specified production limits on p_1 and p_2, that is, the tolerance limits, which vary about a specified nominal value. As can be seen, the original value of the nominal places most of the rectangle outside of the acceptable performance region. The resulting design would have a low yield. By adjusting the nominal to the center of the feasible design region, nearly all of the assemblies will perform satisfactorily.

The second method of increasing yield is by variance reduction, that is, by tightening the tolerances, as shown in Fig. 6b. The new, tighter tolerances place all of the produced assemblies inside the acceptable region. Of course, tighter tolerances are more costly to produce. But the increased cost may be partially offset by the reduction in waste and rework. The optimum tolerances may be found by minimizing the overall cost of tight tolerances, waste and rework [Spence & Sion 1988].

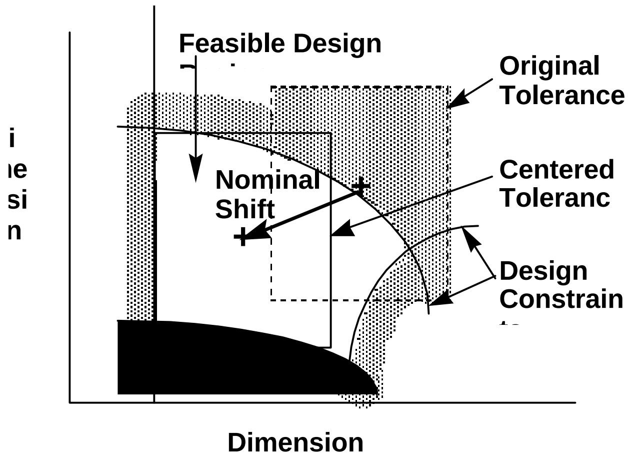


Fig. 6a. Design

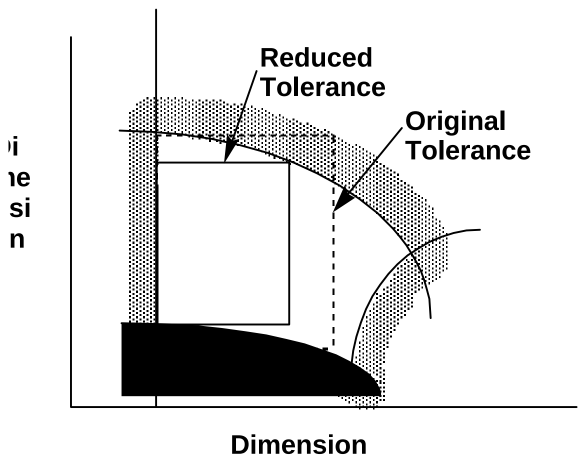


Fig. 6b. Variance

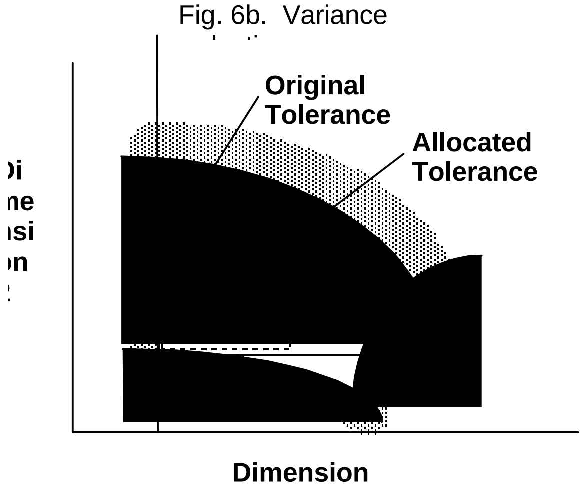


Fig. 6b. Variance

Fig. 6. Illustration of design improvement methods.

7.2 Research in Tolerance Allocation

Tolerance allocation is a **design** function. It is performed early in the product development cycle, before any parts have been produced or tooling ordered. It involves first, deciding what tolerance limits to place on the critical clearances and fits for an assembly, based on performance requirements; second, creating an assembly model to identify which dimensions contribute to the final assembly dimensions; third, deciding how much of the assembly tolerance to assign to each of the contributing components in the assembly. Fig. 6c shows the tolerance on dimension 2 reduced, allowing an increase in tolerance on dimension 1.

Tolerance analysis, on the other hand, is a **production** function. It is performed after the parts are in production. It involves first, gathering data on the individual component variations; second, creating an assembly model to identify which dimensions contribute to the final assembly dimensions; third, applying the measured component variations to the model to predict the assembly dimension variations.

A defective assembly is one for which the component variations accumulate and exceed the specified assembly tolerance limits. The yield of an assembly process is the percent of assemblies which are not defective. In tolerance analysis, component variations are analyzed to predict how many assemblies will be in spec. If the yield is too low, rework, shimming, or parts replacement may be required. In tolerance allocation, an acceptable yield of the process is first specified and component tolerances are then selected to assure that the specified yield will be met.

Often, tolerance design is performed by repeated application of tolerance analysis, using trial values of the component tolerances. However, a number of algorithms have been proposed for assigning tolerances on a rational basis, without resorting to trial and error. Several are listed in Fig. 7.

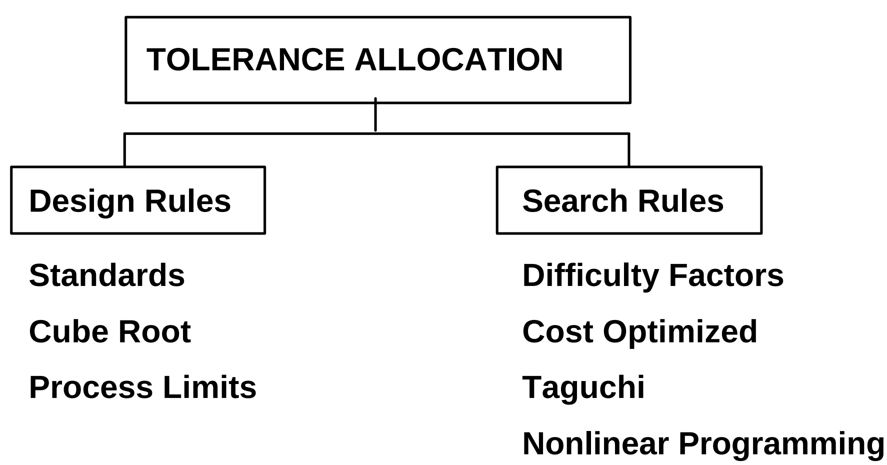


Fig. 7. Tolerance allocation methods.

7.2.1 Proportional Scaling.

By this procedure, initial values of the component tolerances are selected, substituted into the assembly tolerance sum equation, then scaled proportionally so the sum equals the assembly tolerance limit. Initial tolerance values may be selected from charts of tolerance capabilities for specified processes, from design rules, standards, etc.[Mansoor 1963, Chase & Greenwood 1988, Bjorke 1989].

A variation on this method adds flexibility by specifying weight factors to certain component tolerances so those components will receive a greater allocation of the available tolerance [Harry & Stewart 1988].

7.2.2 Cube Root of the Nominal.

This method is based on the rule-of-thumb that the difficulty in obtaining a specified tolerance increases as the cube root of the nominal size of the part . The rule is the basis for the early tolerance standards for cylindrical fits.[Fortini 1967]. The procedure is to select initial tolerance values equal to the cube root of the nominal, substitute into the assembly tolerance sum equation, then scale proportionally. The resulting tolerances will each be proportional to the cube root of their nominal size [Chase & Greenwood 1988].

7.2.3 Difficulty Factors.

This is an extension of the cube root method, with more categories of difficulty, such as: size, shape, material, process, etc., where each category refers to a property affecting the cost of producing a tolerance. The designer assigns a difficulty factor to each component tolerance based on nominal size, then assigns another factor to each component based on shape (inside dimension, outside dimension, etc.), and repeats this process for each category, writing the factors in a table. The difficulty factors for each component dimension are summed and used as weight factors in the tolerance sum equation to drive the allocation [Fortini 1967, 1985].

7.2.4 Minimum Cost.

If an empirical function of cost-vs-tolerance (or process capability) can be obtained for each dimension in the assembly sum, then an optimization algorithm may be used to systematically search for the combination of component tolerances which results in the least overall production cost. Numerous researchers have proposed different search algorithms and different forms of empirical cost functions, as summarized in Table 2.

Table 2. Proposed Cost-vs-Tolerance Models

	Cost Model	Method	Author
Linear	**A - B T**	Linear prog	Edel & Auer [1965]
Reciprocal	**A + B/T**	Lagrange mult	Chase & Greenwood [1988]
		Nonlin prog	Parkinson [1985]
Reciprocal Squared	$\mathbf{A + B/T^{2}}$	Lagrange mult	Spotts [1973]
Reciprocal Power	$\mathbf{A + B/T^{k}}$	Lagrange mult	Sutherland & Roth [1975]
Multi/Recip Powers	$\mathbf{B/T^{k_i}}$	Nonlin prog	Lee & Woo [1990]
		Lagrange mult	Bennett & Gupta [1969]
		Lagrange mult	Chase et al. [1990]
		Nonlin prog	Andersen [1990]
Exponential	$\mathbf{B\ e^{-mT}}$	Lagrange mult	Speckhart [1972]
		Geom prog	Wilde & Prentice [1975]
		Graphical	Peters [1970]
Expon/Recip Power	$\mathbf{B\ e^{-mT}/T^{k}}$	Nonlin prog	Michael & Siddall
[1981,1982]			
Piecewise Linear	$\mathbf{A_i - B_i\ T_i}$	Linear prog	Bjork [1989], Patel [1980]
Empirical Data	**Discrete points**	Zero-one prog	Ostwald & Huang [1977]
		Combinatorial	Monte & Datseris [1982]
		Branch & Bound	Lee & Woo [1989]

The constant coefficient **A** represents the fixed costs, such as tooling, setup, prior operations, etc. The **B** term represents the cost of producing a single component dimension to a specified tolerance **T**. All costs are calculated on a per part basis.

7.2.5 Minimum Cost with Process Selection.

Optimization procedures have been extended to not only find the least cost set of tolerances, but to also select the least cost process from a set of alternative processes for each dimension for the assembly. That is, the computer can decide which process is the most economical to produce each part dimension while considering the tolerances of all of the parts and their cost interactions [Ostwald & Huang 1977, Lee & Woo 1989, Chase et al. 1990].

7.2.6 Minimum Cost – Complex Assembly Models.

Optimization procedures may also be applied to complex assemblies defined by 2-D or multiple vector loops, as described in the next section. Further extensions have been studied as follows:

Nonlinear assemblies   Lee & Woo[1990]

2-D assemblies: Sutherland & Roth[1975], Monte & Datsaris[1982], Parkinson[1985], Andersen[1990]

Multiple loop assemblies: Bennett and Gupta[1969], Lee and Woo[1990], Andersen [1990]

Process mean Shifts: Andersen[1990]

Non-Normal distributions: Parkinson[1985]

7.3 Sensitivity Analysis

The third area of design improvement stems from examining the tolerance sensitivities, which are the partial derivative terms appearing in the tolerance accumulation expressions of Eqs. 1 through 4. The tolerance sensitivity tells the designer which assembly parameter variations have the greatest effect on the critical assembly features. Listing the parameters and their corresponding sensitivities in order of decreasing magnitude reveals which components to focus on for design improvement. Alternately, one could list the product of the sensitivities and their corresponding tolerances in descending order and also calculate the percent contribution made by each to the assembly resultant. Then, starting with the largest contributor, the designer could try to decrease the overall variation by tightening tolerances on the most sensitive components or decrease the overall cost by loosening the tolerance on the least sensitive components [Eaton 1975].

Sensitivity reduction is another approach in which the sensitivity itself is reduced by moving the nominal values to a less sensitive portion of feasible design space. Fig. 8 illustrates this method. In the figure, the contour lines represent lines of constant assembly performance. Closely spaced contours indicate a region of high variability in performance. By moving the nominal design from a region of high variability to a region of low variability, as shown in the figure, the design is made insensitive, or robust, to manufacturing variations. A systematic procedure for accomplishing this is the popular Taguchi method developed by the well-known Japanese expert on quality control [Taguchi 1986, Kacker 1986, Byrne & Taguchi 1987, Taguchi et al. 1989].

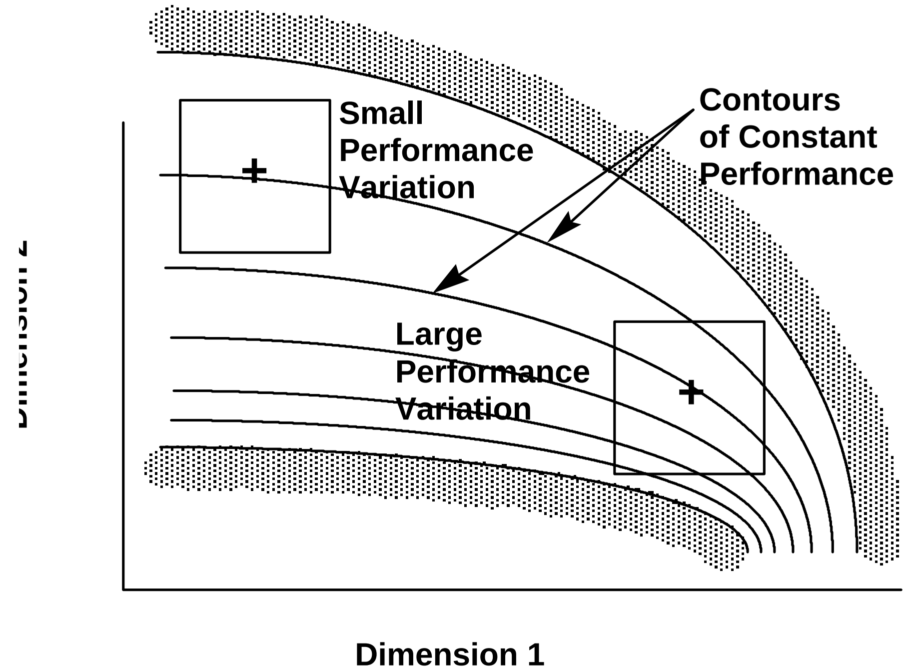


Fig. 8. Sensitivity reduction by shifting the nominal values.

A fundamental element of the Taguchi method is the formulation of a Quality Loss Function which quantifies the cost of deviating from the target value of a given design parameter. The loss function is expressed as a parabola, with minimum cost at the target value and increasing with the square of the deviation. It can include the full spectrum of costs, including inferior performance, increased rework and warranty costs, dissatisfied customers and lost market share. Taguchi also includes the "loss to society", if it can be quantified [Taguchi 1986].

7.4 Probabalistic Design and Design Optimization

Probabalistic design may be considered to be an extension of tolerance analysis methods to include consideration of the variational effects of both geometric and engineering parameters on design performance. Engineering parameters, such as the limiting strength of a metal or the viscosity of a lubricant, exhibit manufacturing variations which can be characterized by statistical distributions. By applying statistical analysis to the engineering performance equations for stress, lubrication, etc., the variation in critical performance resultants can be predicted [Haugen 1980, Mischke 1989].

Design optimization is a mathematical method for improving a design by applying linear or nonlinear programming techniques to search systematically for the minimum of an objective function. The objective function is derived from the engineering model and expresses some critical performance parameter which is to be minimized, such as the weight of a structure or cost of a fluid distribution system.

Often tolerance analysis is performed after the design is essentially complete and all nominal values for the design have been determined. However, by considering the effects of tolerances during the selection of the nominal values of design variables, it is possible to develop a "robust" design that is more tolerant of variation. Developing a robust design by judicious selection of variable nominal values is an important part of the Taguchi philosophy.

Taguchi's methodology develops a model of the design problem by direct experimentation. However, when a computer model of the design exists, an appropriate means of developing a robust design is through nonlinear optimization techniques.

These methods can be used at two levels. The first level is to select design variable values such that the design remains feasible, i.e. will still function properly, despite variations arising from tolerances. The basic approach here is to calculate the variation caused by tolerances using either a first order Taylor's series (Eq (1)) or through Monte Carlo simulation. The transmitted variation is then subtracted from the allowable values of the constraints, causing a shift of the optimum design into the feasible region--as shown in Fig. 9. A good review of applications at this level is given by Eggert (1990).

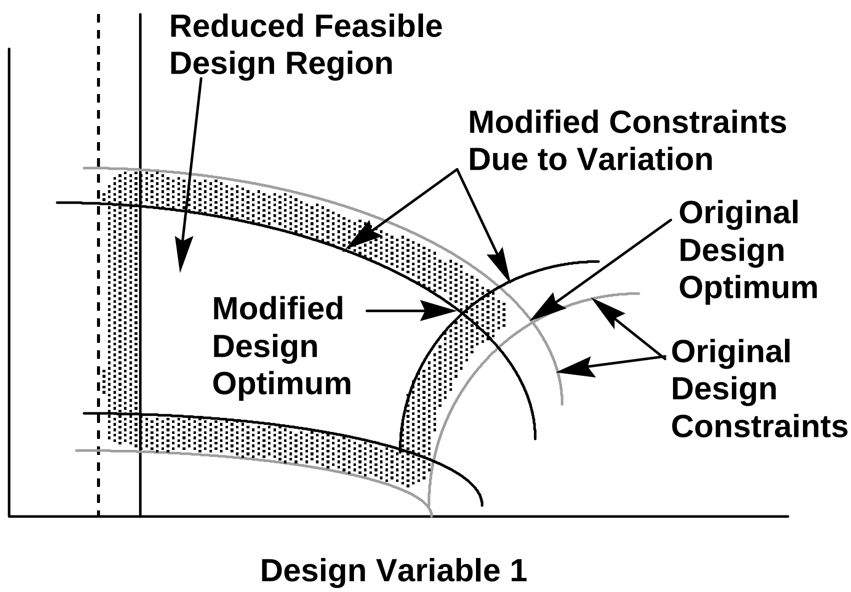


Fig. 9. Change in optimum and decrease in feasible region to insure design will remain feasible to variation caused by tolerances.

The second level is to explicitly consider the variation from tolerances as an objective or constraint in the problem. At this level the designer seeks to actively control variation by either minimizing it as one of the objectives in the design problem or by constraining the design to have variation less than a value specified by the designer. Active control of variation can be computationally expensive since it requires second derivatives of model equations. A good discussion with examples of this level is given by Parkinson et al. (1990).

8 Summary

There is probably no other design improvement effort which can yield greater benefits for less cost than the careful analysis and assignment of tolerances. Tolerancing provides a common meeting ground for engineering and production personnel where effective communication can assure that their competing requirements are met in the most economical way, with the greatest customer satisfaction.

In the foregoing discussion, a number of research areas have been surveyed where significant progress is being made. As a result of current research, powerful new design tools are becoming available which incorporate improved methods for predicting the effects of manufacturing variations on engineering performance and production quality. The effective application of these concepts will assist manufacturing enterprises in competing in the worldwide marketplace.

This literature survey has been so broad in scope that only a few papers have been referenced in each area to permit some tutorial descriptions. We apologize for any papers which may have been passed over in the selection process. A more complete bibliography is available from the authors on request [Chase 1991].

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