Integrating Geometric Form Variations into Tolerance Analysis of 3-D Assemblies

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ABSTRACT

Tolerance analysis is a vital part of any design project since tolerances directly affect cost, quality and performance. Being able to determine the probability of successfully assembling parts and meeting engineering requirements before any parts are manufactured is crucial to the success of a product in today's highly competitive market place. Having the ability to perform tolerance analyses on the computer using a CAD model data base can assure an efficient and accurate tolerancing effort.

There are three main sources of variation in mechanical assemblies: 1) Dimensional, 2) form or feature, and 3) kinematic variations. Form variations arise from variations in shape, orientation or location as described by the geometric dimensioning and tolerancing standard, ANSI Y14.5M. Form variations can have a significant effect on an assembly, since they can accumulate statistically and propagate kinematically the same dimensional variations.

The emphasis of this project is to expand the current computer-aided modeling system to accurately utilize form tolerances as defined by the ANSI Y14.5 standard and exchange this information with a 3-D tolerance analysis package. With all three variation sources accounted for, 3-D tolerance analysis can now be performed entirely using the computer.
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Chapter 1

INTRODUCTION

1.1 Motivation

All manufactured parts in a design have variations introduced by manufacturing process fluctuations and material imperfections. Depending on the application of the design assembly these variations can have costly, and sometimes dangerous, effects. However, the process of evaluating the variations and how they accumulate or "stack up" can be a very drawn out, tedious affair, requiring a knowledge of kinematics, engineering math, statistics and manufacturing processes. Even for the simplest of design assemblies many man-hours and even days can be spent in analyzing the design so the designer can send the design to be manufactured with confidence that the assembly will fit properly and be within the tolerances specified. Unfortunately, many designers choose to scale back their tolerance analysis efforts or eliminate them entirely because of the time and cost involved or because they are unfamiliar with the techniques of tolerance analysis.

The use of properly assigned tolerance specifications can translate into a higher quality product that costs less, which is the main goal of most designers. The problem is being able to assign the proper tolerances in the proper locations. "Loosening" the tolerance specifications may simplify the manufacture of the part(s), but it increases the risk of rejections because the parts do not assemble. This translates into higher production costs and possible redesigns. "Tightening" the tolerance specifications will guarantee the parts will assemble properly, but costs of manufacturing to such tight tolerances may sky-rocket. Therefore, there must be an optimal assignment of tolerances, as shown in figure 1.1, that will yield the highest quality product for the least amount of expense.
Figure 1.1 Cost vs. Tolerances Relationship

The cost curves of figure 1.1 are a general representation of the affect of tolerances on the cost of a product. Two cost curves are shown. One showing the increase in production cost as tolerances are tightened, the other showing the increase in cost of rejects as tolerances are loosened. The optimum tolerance is the value which minimizes the total cost. Similar curves could be generated for each component to obtain the total of tolerances for an assembly.

The tolerancing process, as stated above, can become very time-consuming, complex and prone to error. Efforts are currently underway to automate the tolerance specification process in order to assist the designer. With the increasing use of computers and Computer-Aided-Design, Computer-Aided-Manufacturing and Computer-Aided Engineering (CAD/CAM/CAE) software, it seems logical to integrate tolerance analysis models in the CAD process. By utilizing the same CAD model the designer created, complete with form and feature tolerances, the assembly tolerance model can be created in less time, with fewer errors, and less manual data entry.

Also, by automating much of the mathematical modeling and statistical manipulations, the engineer's reluctance to do tolerance analysis can be abated. Using specialized graphical and analytical tools, the designer can quickly see what features
have the greatest influence in tolerance propagation. The designer can then make an intelligent decision of which tolerances need to be "tight" and which ones can be "loosened". With the computer doing the analysis, this could be accomplished in minutes instead of days.

1.2 Definitions

In order for the reader to fully understand the contents of this thesis a few definitions need to be introduced.

A feature is a face, edge, vertex, centerline, or centerplane. In regards to a size tolerance, a pair of parallel planar surfaces is also referred to as a feature [Gangoiti, 1991]. Size and form tolerances are applied to the feature itself, and orientation and location tolerances are applied relative to one or more datums.

A tolerance is a constraint on the size, the orientation, the location, or the form of a part feature. A designer specifies tolerances in an effort to control the variations of the part or assembly. The tolerances specify permissible variations in the geometry of the part.

A size tolerance is a tolerance applied to the size of a feature such as the length of an edge or the radius of a sphere.

A form tolerance applies to the allowable form variation of a feature, such as flatness or cylindricity. The form of a feature is controlled by the size, location, or orientation tolerance. The form tolerances are used when tighter control is required to meet functional requirements. Therefore, the form tolerance is in addition to the size tolerance. A standard system of form tolerances is defined be ANSI Y14.5M [1982].

Kinematic variations are the small adjustments possible between two mating parts defined by the degrees of freedom of the joint contact between the two parts.

Component tolerances are the allowable variations of the features of a single part. Component tolerances are only critical when they contribute to a tolerance stack which effects an assembly spec.

Assembly tolerances are the allowable variations of an entire assembly.

Engineering design specs are almost always applied to assembly tolerances.
1.3 The Importance of Form Tolerances

The three principle sources of variation in mechanical assemblies are 1) size, 2) form and 3) kinematic. It is well known that size tolerances are vital to the production of the part and especially the assembly as a whole. There seems to be less importance put on form tolerances as there is for size tolerances. Form variations are generally an order of magnitude smaller than dimensional variations, but they can accumulate and propagate the same as dimensional variations. In some cases, the resulting rigid body effects can be significant. Since variations in form and feature of a part can occur independently of size tolerances, form tolerances cannot be neglected if the design must assemble properly. On this premise, a proper tolerance analysis must therefore include form tolerances.

1.4 The CAD Model

The CAD model should take into account all of these variation sources and tolerancing limits. In order to provide this tolerancing information, geometric information about the assembly is required. By allowing the designer to enter this information directly into the CAD model, this important information is always available for analysis. The ability to enter this information in a graphics environment also lends greater ease in viewing and picking appropriate tolerances, especially form tolerances. An example of a 2-D graphical model of an assembly suitable for tolerance analysis is shown in figure 1.2.
Figure 1.2 Example of Graphical Model of an Assembly

This model uses vectors and kinematic joints to simulate manufacturing variations in a mechanical assembly. A system based on this model representation, linked to a CAD system, is currently under development at Brigham Young University. This system overlays a vector model of an assembly on a CAD solid model and contains the dimension and tolerance data required for assembly tolerance analysis.
1.5 Research Objective

The purpose of the research presented in this thesis is to define models for 3-D form variations and to extend current assembly tolerance models to include geometric form and orientation variations. This will include the ability to analyze multiple part assemblies using both statistical and worst case analysis.

1.6 Research Approach

Three-dimensional assembly tolerance models will be developed for each type of form variation defined by ANSI Y14.5 and applied to the tolerance analysis of 3-dimensional assemblies. Form variations only contribute to assembly variations through the points of contact between mating parts. As will be described later, the models for form variation are dependent upon the kinematic joints and joint axes through which they propagate. A complete description of form tolerances must include engineering models for each ANSI Y14.5 feature variation applied to each 3-D kinematic joint type. These models for form variations will become part of a system for creating engineering models of assemblies graphically on a CAD system. The data format used for data exchange between the CAD system and the analysis package will be updated to allow for form variations. A comprehensive system for statistical tolerance analysis which includes dimensional, kinematic and form variations in assemblies will then be possible. This work, however, will be limited to demonstrating analysis procedures for form tolerances. Coding for the analysis program and testing is presently being carried out by research associates.

1.7 Thesis Overview

This chapter has introduced the research to be presented in this thesis. Chapter 2 discusses important background information regarding research related to computer-aided tolerance modeling and analysis. Chapter 3 presents the 3-dimensional models developed for each type of form variation defined by ANSI Y14.5 and applies them to a selection of kinematic joints. Chapter 4 explains the theory and equations involved with the research. Chapter 5 presents several example assembly models and compares the analysis results with and without form tolerances. Conclusions are presented in Chapter 8 along with recommendations for future research.
Chapter 2

BACKGROUND

The idea of automated tolerancing has given rise to a great deal of research in that area over the last decade. Progress is being made in many areas towards automating the tolerance analysis process. Several approaches have been introduced that address the tolerancing problem, will be discussed in this chapter. As they all have positive and negative features, there seems to be disagreement on which is the best. One thing that has emerged from the research is the seemingly unanimous opinion that the automated tolerancing solution must include the CAD system used by the designers. This chapter will focus on current research in tolerance modeling.

2.1 Assembly Tolerance Modeling Requirements

In order for an assembly modeler to properly represent a design assembly for tolerance analysis, it must account for the following effects:

1. Rigid body translation and rotation
2. Propagation of form and feature variations
3. Propagation of kinematic adjustments
4. Dimensional variations
5. Tolerance accumulation

The effect of variations of position and orientation of parts in an assembly can have an adverse affect on other parts in the assembly. These effects of other parts can be seen in terms of translations and rotations [Larsen 1991; Marler 1988] and is termed rigid body motion. Other variations can manifest themselves in forms of manufacturing variations and how part surfaces or other features vary from their ideal design. This not only suggests the need for representing form and feature variations but representing their effects as well.

The modeling of kinematic adjustments is very critical and demands special treatment. There has been much work done in modeling these variations [Larsen, 1991;
Robison, 1989; Marler, 1988]. More recent work has been done in the area of modeling the form and feature variations as well as kinematic adjustments [Goodrich, 1991]. Goodrich developed a method of including form tolerances in the solid model on CATIA. Using CATIA's programming interface, he provided the ability for engineering designers to overlay tolerance information directly into the CAD model. The ability to enter tolerance information is now in place but, there is still not a way to automatically extract the data. Goodrich also developed a mathematical representation of form tolerances that can be used in worst case and statistical analyses of an assembly tolerance models. These form tolerances need to be looked at extensively to determine the types of variations introduced by each form tolerance according to which part contact joint is involved.

2.2 Research Review

There are three major approaches to modeling assemblies for tolerance analysis. They are:

1. Solid model
2. Variational geometry
3. Vector-Based

Turner [1987, 1990] and his associates have proposed using a solid model created on a CAD system which serves as the assembly function. Small changes can be simulated and their effects will propagate realistically, provided each part is located relative to its adjacent parts and provided that kinematic adjustments are permitted. Sensitivities are obtained by making small changes in each of the model variables, measuring the resultant change in the component dimensions and assembly resultants and computing. However, solid modelers are CPU intensive and changing a single parameter for a sensitivity calculation requires regeneration of the entire CAD geometry. A detailed model of an assembly may have thousands of model parameters, resulting in a substantial wait for a complete sensitivity calculation on all but the most powerful computers.

A number of researchers are taking an axiomatic approach to 3-D tolerance representations in solid models. Requicha represents the model variations as a pair of "offset boundaries," of offset surfaces, which bound each ideal surface. The set of
offset boundaries form a tolerance zone which bounds the entire part [Requicha, 1983, 1986]. A similar definition creates a "virtual boundary" formed by taking into account the combined effects of all applicable size and form tolerances [Jayaraman and Srinivasan, 1989, Srinivasin and Jayaraman, 1989]. Several problems remain to be resolved, including: potential conflicts with existing standards, incorporation of statistical models and the lack of kinematic assembly interactions.

Another approach is the use of variational geometry. It requires the formulation of analytical equations describing the geometric relationships which must be maintained in an assembly. Constraints such as perpendicular surfaces or surfaces in sliding contact are defined in terms of dimensional parameters. If the design is modified, the system of equations may be solved to adjust the free variables in keeping with the constraints. The advantages of this method are the ease of design iteration and the realistic propagation of manufacturing variations by kinematic adjustments. However, the resulting system of nonlinear equations can become very large and must be solved simultaneously. Also, geometric form and feature tolerances must still be taken into account [Light and Gossard 1982, Gossard et al. 1988, Chung and Schussel 1990].

A related method based on the actual assembly process has been proposed by Kim and Lee [1989]. An assembly model is created by having the designer specify the mating conditions between the various parts in the assembly. The type, location, and orientation information of the mating conditions (kinematic joints) can be derived automatically from these specified conditions. The idea of automating the process is definitely in the right direction for design automation. There are some limitations, however, such that the correct mating condition (joint) is not always selected. Also, there is only a limited set of kinematic joints defined for use.

Richard Robison has proposed a method of representing tolerance information in a solid model [Robison, 1989]. This is a parametric space type model and is composed of a vector-based method for modeling 3-D mechanical assemblies. It uses vectors to represent dimensions between critical part features and includes a set of kinematic joint types to represent mating conditions between parts at points of contact. This method also includes rules for identifying a valid set of vector loops to make sure the tolerance model is complete. It is independent of the solid modeler and can be integrated with a tolerance analysis package to solve for the tolerance sensitivity on the desired dimensions or clearances. This allows for form and feature controls to be included in the model which has been viewed as extremely important. One major advantage of 3-D
vector models of assemblies over solid models is that the geometry is reduced to only those parameters required to perform a tolerance analysis. Sensitivity analysis is much simpler. It is very efficient computationally and well suited to design iteration. Also, engineering designers are already familiar with describing engineering models with vectors and kinematics.

Comparisons of the three approaches to assembly models discussed above are summarized in Table 2-1.

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid model</td>
<td>Created on CAD system. Changes propagate realistically.</td>
<td>CPU intensive</td>
</tr>
<tr>
<td>Variational geometry</td>
<td>Tolerance information adjusts with geometry</td>
<td>Equations nonlinear and must be solved simultaneously. Geometric form and feature tolerances not included.</td>
</tr>
<tr>
<td>Vector-based</td>
<td>Independent of solid modeler. Allows for form and feature controls. Efficient computationally. Can be integrated with a tolerance analysis package.</td>
<td>None</td>
</tr>
</tbody>
</table>

2.3 CAD Based Assembly Modeler

The proliferation of CAD systems in the design and engineering world has provided for vast amounts of electronically stored part and assembly drawing data. Because of this electronic data base, we have readily available to us stored information of each of those drawings that can be very useful in designing and manufacturing an assembly. However, even though CAD gives us a graphical representation of the assembly and how the individual parts interface, it is not enough for further applications [Srikanth et al., 1990; Shah et al., 1990]. Many researchers recognize the necessity for
a system to provide assembly information as well as part feature, dimension, and tolerance information.

A tolerance modeler should consist of parts, datums, joints and loops overlaid on the existing CAD geometric assembly model [Larsen, 1991; Marler, 1988]. There are definite advantages to using existing geometric data. The biggest advantage is the fact that the use of existing data avoids duplication of effort in providing that information and it also avoids the introduction of data entry errors. Another advantage is it allows for a graphical input rather than text input. ElMaraghy [ElMaraghy, 1991] emphasizes the fact that having a tolerance modeler and analysis tool available at the design prototyping stage "is of paramount importance as it can significantly reduce the design development cost, increase its efficiency, reduce the time cycle between product design and manufacture, and improve the quality of resulting design." The results of meaningful evaluation and analysis using such tools can provide for valuable suggestions to improve the design.

Martino and Gabriele [1989] point out that in addition to preserving relationships between features, a tolerance modeler can make tolerance analysis easier because the model can potentially be used to automatically derive the relationships between features and parts that describe how tolerances will effect critical clearances or dimensions in the assembly. If tolerances were tied to dimensions it would allow the tolerance modeler to be updated when geometry is changed. Also, with geometric tolerance information available, it is possible to automate part inspection for quality assurance checking [Goodrich, 1991].

Whichever model is used, it should accurately reflect current practice and standards and should also allow implementation into a CAD modeler or computer-aided-analysis. The CATS system for tolerance analysis integrated with the CATIA 3-D geometric modeler is illustrated in figure 2.1.
The research for this thesis is based on the parametric space type model proposed by Robison and further work by Goodrich. The mating conditions defined in the tolerance model can be extracted automatically and entered into a tolerance analysis package via a neutral file. The CATS (Computer Aided Tolerance Selection) analysis package, developed by Brigham Young University, takes the information passed to it and makes small perturbations to the part dimensions. It then determines the accumulative effects of the variations statistically. This analysis package allows for many sources of variation to occur simultaneously.

2.4 Solid Modeling Challenges

There are some limitations of solid modelers which create problems for maintaining this information when changes are made to the model. Figure 2.2 illustrates the process involved with creating surface geometry using CATIA.
As shown in figure 2.2, elements are constructed initially as faceted geometry which have the advantage of being easy to compute and fast to display. The disadvantages are faceted geometry is not as accurate and there are no surfaces to relate tolerances to. Surface geometry is very accurate and provides the desired surfaces for relating tolerance information to. However, surface geometry is slow and expensive and very tedious to edit. Also, CATIA does not provide associativity for either faceted or surfaced geometry. Hence, any changes to the geometry following tolerance input requires re-inserting the tolerance graphics and information. Surface geometry is even more difficult to work with since all changes must be done on the faceted geometry and then converted to surface geometry using CATSOL. This means all tolerance information and geometry is lost and must be re-entered in its entirety.

2.5 Graphical Interface

The vector loop method of including tolerance information in the solid modeler [Goodrich, 1991; Robison, 1989] provides for good visual feedback to the user. Figure 2.3 shows an example of stacked blocks and how the tolerance model looks to the designer.
The loops and joints contain the needed information to provide for an accurate tolerance analysis. The interface between the user and the model data base is accomplished by using the Application Programming Interface (API) available with CATIA. This is a programming utility that provides "hooks" into the solid model data base. A programmer can take advantage of available function calls to gain access to the data base and extract information, enter new information, change existing information and create or delete graphical entities in the solid model. The API utility has allowed development of custom "pop-up" windows for data manipulation. It also provides the ability for including custom FSD files used in creating personalized side menus. Figure 2.4 shows the CATIA.CATS menu structure.
### CATS Menu Structure

<table>
<thead>
<tr>
<th>DISPLAY Toggles</th>
<th>OFF/SWITCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARTS DRF</td>
<td>ON/OFF</td>
</tr>
<tr>
<td>FTR_DTM</td>
<td>ON/OFF</td>
</tr>
<tr>
<td>D OF</td>
<td>ON/OFF</td>
</tr>
<tr>
<td>JOINTS</td>
<td>ON/OFF</td>
</tr>
<tr>
<td>FTR_CTRLs</td>
<td>ON/OFF</td>
</tr>
<tr>
<td>LOOPS</td>
<td>ON/OFF</td>
</tr>
</tbody>
</table>

**Symbol Size**
2.0 mm

### PART DATUMS
- RECT – Rectangular Datum
- CYLCEN – Centerline Datum
- SPHCEN – Sphere Center

### KINEMATIC JOINTS
- RIGID – Rigid (weld, bolt)
- REVOLU – Revolute
- CYLIND – Cylindrical
- PRISM – Prismatic
- BALL – Ball-and Socket
- PLANAR – Planar Slider
- EDGSLI – Edge Slider
- CYLSLI – Cylindrical Slider
- PNTSLLI – Point Slider
- SPHSLI – Spherical Slider
- CRSCYL – Crossed Cylinders

### CLEARANCE TYPES
- PT-PT – Point-to-Point
- PT-PL – Point-to-Plane
- PL-SF – Point-to-Surface
- PL-PL – Plane-to-Plane
- PL-SF – Plane-to-Surface
- SF-SF – Surface-to-Surface

### FORM/FEATURE CONTROLS
- FLA – Flatness
- STR – Straightness
- CIR – Circularity
- CYL – Cylindricity
- LPR – Line Profile
- SPR – Surface Profile
- PAR – Parallelism
- PER – Perpendicularity
- ANG – Angularity
- CRN – Circular Runout
- TRN – Total Runout
- POS – True Position
- CON – Concentricity

---

Figure 2.4 CATIA.CATS Menu Structure
2.6 Building a Kinematic Model

Figure 2.5 shows the 3-D joint set considered, showing the kinematic degrees of freedom. The joints here were defined to account for a variety of part mating conditions.

<table>
<thead>
<tr>
<th>Joint Type</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid (no motion)</td>
<td><img src="image" alt="Rigid Joint" /></td>
</tr>
<tr>
<td>Prismatic (1)</td>
<td><img src="image" alt="Prismatic Joint" /></td>
</tr>
<tr>
<td>Revolute (1)</td>
<td><img src="image" alt="Revolute Joint" /></td>
</tr>
<tr>
<td>Parallel Cylindrical (2)</td>
<td><img src="image" alt="Parallel Cylindrical Joint" /></td>
</tr>
<tr>
<td>Cylindrical (2)</td>
<td><img src="image" alt="Cylindrical Joint" /></td>
</tr>
<tr>
<td>Spherical (3)</td>
<td><img src="image" alt="Spherical Joint" /></td>
</tr>
<tr>
<td>Planar (3)</td>
<td><img src="image" alt="Planar Joint" /></td>
</tr>
<tr>
<td>Edge Slider (4)</td>
<td><img src="image" alt="Edge Slider Joint" /></td>
</tr>
<tr>
<td>Cylindrical Slider (4)</td>
<td><img src="image" alt="Cylindrical Slider Joint" /></td>
</tr>
<tr>
<td>Point Slider (5)</td>
<td><img src="image" alt="Point Slider Joint" /></td>
</tr>
<tr>
<td>Spherical Slider (5)</td>
<td><img src="image" alt="Spherical Slider Joint" /></td>
</tr>
<tr>
<td>Crossed Cylinder (5)</td>
<td><img src="image" alt="Crossed Cylinder Joint" /></td>
</tr>
</tbody>
</table>

Figure 2.5 3-D Joint Set For Part Mating Conditions

The appropriate joint, determined by the part mating condition, is applied at each part interface in an assembly. The joints must be oriented correctly for the degrees of freedom to be analyzed correctly. The vector loops provide the critical dimensions for the applicable part features as well as the datum path selected associating joints to the two mating part's datum reference frames.

2.7 Datums

M. F. Spotts [1985] identifies the need for a "datum" system or in other words a Datum Reference Frame (DRF). All dimensions applicable for a part should be
referenced from the DRF. Datums determine which dimensions will be included in vector loops. This system should consist of three mutually orthogonal planes for rectangular coordinates. For cylindrical parts, the DRF would be a perfect cylinder butted up against a perpendicular plane.

2.8 Form Variations

The data objects for a tolerance modeler should also include form and feature controls if accurate modeling is the objective. Since the variations in form and feature of a part can affect other part position and orientation. The accumulation of individual form and feature variations has the possibility of greatly affecting the final assembly. Thus the modeler must be able to maintain a relationship between features.

Spotts [1985] points out that datums alone will not guarantee perfectly accurate manufactured parts. The variations introduced by the manufacturing process requires geometric tolerancing to include form and feature tolerances. This is echoed by Shah and Miller [1990] when they state that form, orientation, position, and runout variations must be controlled as well as size variations.

The importance in assessing limits to feature variations is brought up by the fact that there is a ANSI Y14.5M standard. This standard places limitations on the variation of features in terms of their form, location, and orientation [ANSI, 1982]. Figure 2.6 shows the symbols representing the geometric feature controls defined by the ANSI standard.
### Figure 2.6 Symbols for ANSI Y14.5M-1982 Geometric Feature Controls

The ANSI Y14.5M standard has grouped the feature control tolerances into the following five main categories:

1. **FORM** - A form tolerance states how far an actual surface or feature is permitted to vary from the desired form implied by the drawing or CAD model.

2. **PROFILE** - A profile tolerance states how far an actual surface or feature is permitted to vary from the desired form on the drawing and/or vary relative to a datum. Profile is a general case of form that includes other geometric forms in addition to cylinders and planes.

3. **ORIENTATION** - An orientation tolerance states how far an actual surface or feature is permitted to vary relative to a datum.

4. **LOCATION** - A location tolerance states how far an actual size feature is permitted to vary from the perfect location implied by the drawing as related to a datum or other features.

5. **RUNOUT** - A runout tolerance states how far an actual surface or feature is permitted to vary from the desired form implied by the drawing or CAD model during full (360°) rotation of the part on a datum axis. Runout is a composite of form, orientation and location effects.
2.9 Analysis of Assembly Variation

A common method used for tolerance analysis of assemblies is based on Monte Carlo simulation. This method generates a random variation for each part in the assembly based on the tolerance information available and a statistical distribution of the variation for that part. The simulated parts are then "assembled" and a statistical model of how the tolerances relate in the final assembly is created. This method is quite accurate in its analysis, but the number of samples required to obtain an accurate result most often reaches into the hundreds of thousands [Shapiro and Gross 1981]. Advantage of this method is the ability to analyze non-normal distributions and non-linear assembly relationships. Disadvantages are large sample sizes and current systems do not include kinematic constraints, which require an iterative solution to determine the assembly resultants for each assembly. The resultant computation time is excessive.

Another approach to tolerance analysis of assemblies is based on linearization of the vector loop equations and solving for all assembly variables by linear algebraic manipulation. CATS.BYU is an engineering software package developed at Brigham Young University [Chase, 1987]. Available in this package are several tolerance analysis options, including both worst case and statistical analysis of the assembly tolerance, and several methods for allocating tolerances for unspecified components. This package can now accept tolerance information created on a CAD system for use in its analysis process.

The CATS analysis software receives tolerancing information either through an input window in the analysis package itself, or transferred from a graphics modeler in the form of a neutral file. The tolerancing information is then used to determine the dependent variable values. These values are the sensitivities of the kinematic variables in the assembly model when using closed loops, or the sensitivities of the variations at the end of the loop when using open loops.

2.9.1 Closed Loop Analysis

Closed loops give the ability to find solutions to the kinematic variables and how much of an effect they have on the assembly variations as a whole.

After linearization, the vector equations for a closed loop system may be written in matrix notation:

Where \([B]\) and \([A]\) are the sensitivity matrices and \(du\) and \(dx\) are vectors of the associated variations in the kinematic and manufacturing dimensions, respectively.

Rearranging and solving for \(du\) gives,

\[ du = -[B^{-1}A]dx \] (2.2).

Adding form tolerances enhances the equations to give,

\[ dh = [B]du + [A]dx + [F]d\alpha \] (2.3).

Rearranging and solving for \(du\) again leaves,

\[ du = -[B^{-1}A]dx - [B^{-1}F]d\alpha \] (2.4).

Where \([F]\) is the form tolerance sensitivity matrix and \(d\alpha\) are the corresponding form tolerances.

The sensitivities are formed by perturbing each dimensional or kinematic variable by a small amount, calculating the resulting change in the loop equation (\(\delta x, \delta y, \delta z\)) and calculating and storing its sensitivity. Likewise, each rotational variable is perturbed by some small amount \(\delta \theta\) and its sensitivity calculated. After all the sensitivities are found in the vector loop, the variation \((du)\) of any particular dependent dimension can be calculated given the variations or tolerances \((dX_j)\) for the independent dimensions. Equations 2.5 and 2.6 are given for worst case and statistical assembly tolerance analysis respectively [Marler, 1988].

Worst Case –

\[ dU = \sum_{j=1}^{n} \left| \frac{\partial u}{\partial X_j} \right| dX_j \leq T_{asm} \] (2.5)

Statistical –

\[ dU = \sqrt{\sum_{j=1}^{n} \left[ \frac{\partial u}{\partial X_j} dX_j \right]^2} \leq T_{asm} \] (2.6)
Goodrich [1991] and Robison [1989] added to these equations the ability to include form tolerances. The enhanced worst case and statistical forms of equations 2.5 and 2.6 are shown in equations 2.7 and 2.8 below.

Worst Case: \[ dU = \sum \left| \frac{\partial u}{\partial x_j} \right| dx_j + \sum \left| \frac{\partial u}{\partial \alpha_j} \right| d\alpha_j \leq T_{asm} \] (2.7)

Statistical: \[ dU = \sqrt{\sum \left[ \frac{\partial u}{\partial x_j} dx_j \right]^2 + \sum \left[ \frac{\partial u}{\partial \alpha_j} d\alpha_j \right]^2 } \leq T_{asm} \] (2.8)

The \( \alpha \) terms in these equations represent the form variations included in the tolerance model. These terms are added separately because form variations are assumed to be independent of dimensional variations.

The key to determining the affect of form variations on assembly variations is the determination of the sensitivity matrix \([F]\) and the resulting tolerance sensitivities \(B^{-1}F\).

2.9.2 Open Loop Analysis

If an assembly is such that a closed vector loop cannot be found, such as where there is a clearance associated with the assembly, an open loop must then be used.

Open loops provide the ability to solve for the variation sensitivities at the end of the loop \((dh_{OL})\) with respect to the start of the loop [Carr 1992]. The open loop equation is defined as,

\[ dh_{OL} = Cdx + Ddu \] (2.9)

Where \( C \) is the matrix containing the partial derivatives of open loops with respect to independent variables and \( D \) is the matrix containing the partial derivatives of open loops with respect to dependent variables. The \( dx \) and \( du \) matrices are the part tolerances and the kinematic variable tolerances, respectively.
By substituting \( du = -[B^{-1}A]dx \) found in the closed loop equation 2.2, equation 2.9 becomes,

\[
dh_{OL} = Cdx + D[-B^{-1}Adx].
\]

Bringing \( D \) inside the brackets gives,

\[
dh_{OL} = Cdx - D[B^{-1}A]dx
\]

and combining like terms leaves us with the open loop equation,

\[
dh_{OL} = [C - DB^{-1}A]dx
\] (2.10)

The development of the open loop equation that includes form tolerances is very similar. However, instead of substituting \( du = -[B^{-1}A]dx \), you substitute \( du = -[B^{-1}A]dx - [B^{-1}F]d\alpha \). By substituting \( du \) into the general open loop equation,

\[
dh_{OL} = Cdx + Ddu + Gd\alpha.
\] (2.11)

(where matrix \( G \) contains the partials of open loops with respect to form variations \( a \) and \( d\alpha \) contains the feature tolerances. Matrices \( C, D, dx \) and \( du \) are the same as described above), and combining like terms, the final open loop equation becomes,

\[
dh_{OL} = [C - DB^{-1}A]dx + [G - DB^{-1}F]d\alpha
\] (2.12)

In solving for the worst case or statistical solution to the open loop equations, it is important to note that the solution is an \( x, y, z \) positional variation and/or an \( \theta x, \theta y, \theta z \) angular variation as compared to the statistical and worst case solution for closed loops which pertain to kinematic variations.

The worst case method for solving open loop systems is quite similar to the worst case method for solving closed loop systems. The general equation for worst case is given as follows (compare equation 2.7),

\[
\text{Worst Case} - \quad dU = \sum \left| \frac{\partial u}{\partial x_j} \right| dx_j + \sum \left| \frac{\partial u}{\partial \alpha_j} \right| d\alpha_j \leq T_{asm}
\] (2.13)
where $du$ is the $d\Theta_L$ discussed above and $\partial u/\partial X$ and $\partial u/\partial \alpha$ are the matrices $[C - DB^{-1}A]$ and $[G - DB^{-1}F]$ respectively found in equation (2.12). Substituting the terms used in equation (2.12) into equation (2.13) gives us,

$$du = |C - DB^{-1}A|dx + |G - DB^{-1}F|d\alpha$$  \hfill (2.14)

It is also important to note that the form variations are independent of the kinematic variations and are, therefore, additive in equation (2.14).

As the worst case methods in open and closed loops are similar, so are the statistical methods similar. The general equation for statistical solutions is given as follows (compare equation 2.8),

$$dU = \sqrt{\sum \left[ \frac{\partial u}{\partial X_j} dX_j \right]^2 + \sum \left[ \frac{\partial u}{\partial \alpha_j} d\alpha_j \right]^2}$$  \hfill (2.15)

where $du$, $\partial f/\partial X$ and $\partial f/\partial \alpha$ are defined the same as in the worst case analysis above. Making the same substitutions as was done in equation (2.13) results in the following statistical equation,

$$dU = \sqrt{\sum \left| [C - DB^{-1}A] \right|^2 + \sum \left| [G - DB^{-1}F] d\alpha \right|^2}$$  \hfill (2.16)

Again, it must be pointed out that the form variations are independent of the kinematic variations and are, therefore, additive as in equation (2.14).

The final expressions of the statistical Root Sum Squared (RSS) or Worst Case (WC) methods have a very important property associated with them. These final expressions are the results of looking at all possible combinations of assembly tolerances as opposed to a single assembly combination that other methods use such as Monte Carlo simulation.
Chapter 3

KINEMATIC JOINTS AND FORM TOLERANCES

3.1 Effect of Geometric Form Variations

Process variations can be introduced into an assembly from a number of sources, such as error in the location of stops or in the calibration of a milling machine; the error in the positioning of the cutting tool on a lathe; or just simply wear of the tools themselves. These examples describe a source of variation in the size or location of the feature of a part as compared to perfect or nominal size. Another source of variation has to do with the form of the feature as compared to perfect form, regardless of its size. This latter type of variation (form variation) is the focus of this section.

Form and feature controls as defined by the ANSI Y14.5M-82 standard allows the designer to apply additional limitations on the variation of form or orientation of a feature on the part. These limits are in addition to the size tolerances. This allows necessary tolerance information to be added to an individual part feature not provided by size tolerances alone. The tolerances are usually associated with the function of an individual part feature or its relationship to another feature on the same part. Lowell W. Foster [Foster 1986] identifies some important conditions that indicate when it is appropriate to use geometric form tolerances:

1. When part features are critical to function or interchangeability

2. When functional gaging techniques are desirable

3. When datum references are desirable to ensure consistency between manufacturing and gaging operations.

4. When computerization techniques in design and manufacture are desirable

5. When standard interpretation or tolerance is not already implied

Geometric variations may or may not affect the size or orientation of assembly features. This depends on the Geometric (form) tolerance selected, the condition of contact between the mating parts, the width of the tolerance zone, and the size of the contact
area of the mating part. Figure 3.1 shows the effects a flatness geometric form variation can have on 3-D mating surfaces.

![3-D Feature Variations Diagram]

**Figure 3.1 3-D Effects of Form Variation**

The manner in which surface variations are propagated through this assembly depends on the type of surface contact. Consider a cylinder resting on a plane. If the plane exhibits waviness or flatness variations, the location and orientation of the cylinder will be affected. Looking down the axis of the cylinder (z-axis), it can be seen that the cylinder could rest in a peak or down in a valley of the surface, producing a translational variation normal to the plane. Also, looking perpendicular to the z-axis (x-axis), a rotational variation about the x-axis is produced when one end of the cylinder rests on a peak and the other end lies in a valley.

Similarly, variations in the form of the cylindrical surface can produce variations in its location and orientation. If the cylinder is lobed, it may be assembled resting on a lobe, as shown in figure 3.1, or between lobes. Looking down the axis of the cylinder, the this would produce a translational variation normal to the plane at the point of contact. On the other hand, if the cylinder were tapered, a rotational variation would be produced about the x-axis.
Thus, we see that two mating surfaces can introduce several modes of variation into an assembly, causing each assembly to be slightly different from the next.

The mathematical representation for the propagation of variations through the assembly is based on matrix multiplication using rotation and translation matrices. Shown below is a rotation matrix representing the \(i^{th}\) contact joint rotating about the \(j^{th}\) axis and a translation matrix for the same \(i^{th}\) contact joint with a translation along the \(j^{th}\) axis.

\[
\mathbf{R}_{i}(\phi_j) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \phi_j & -\sin \phi_j & 0 \\ 0 & \sin \phi_j & \cos \phi_j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{T}_{i}(\Delta \mathbf{J}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \Delta X & \Delta Y & \Delta Z & 1 \end{bmatrix}
\]

To calculate sensitivities, small perturbations \(\delta \phi \) or \(\delta J\) would be introduced into the respective rotation and translation matrices as shown below.

\[
\mathbf{R}_{i}(\phi_j+\delta \phi_j) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\phi+\delta \phi) & -\sin(\phi+\delta \phi) & 0 \\ 0 & \sin(\phi+\delta \phi) & \cos(\phi+\delta \phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{T}_{i}(\Delta \mathbf{J}+\delta \mathbf{J}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \Delta X+\delta X & \Delta Y+\delta Y & \Delta Z+\delta Z & 1 \end{bmatrix}
\]

Assemblies are modeled using vector loops with each vector being represented by three rotation matrices and one translation matrix, i.e. \([\mathbf{R}][\mathbf{R}][\mathbf{R}][\mathbf{T}]\). The equation for the assembly resultant uses the above matrices as shown,

\[
\mathbf{U}_i = [0 \ 0 \ 0 \ 1][\mathbf{R}_{1x}][\mathbf{R}_{1y}][\mathbf{R}_{1z}][\mathbf{T}_1] \ldots [\mathbf{R}_{ix}][\mathbf{R}_{iy}][\mathbf{R}_{iz}][\mathbf{T}_i] \ldots = [0 \ 0 \ 0 \ 1]
\]

The example in figure 3.1 shows that a translational (T) variation, a rotational (R) variation, or a combination of both (RT) can be defined for each form tolerance-joint combination. In this thesis \(T_x\), \(T_y\), \(T_z\) will be used to denote translation along the x, y, or z axis respectively. In the same context \(R_x\), \(R_y\), \(R_z\) will be used to denote rotation about the x, y, or z axis.
3.2 Kinematic Joints and Geometric Form Tolerances

This section will attempt to define the effect each form tolerance has on each kinematic joint. The twelve different joint types will be looked at individually in relation to the eleven form tolerances used. The kinematic (K) degrees of freedom and form (F) degrees of freedom will be discussed.

Each kinematic joint has its own unique set of local axes to aid in determining the degrees of freedom and direction of variations of those degrees of freedom, introduced by that particular joint and any form tolerance associated with it.

In three-dimensional problems, there are six possible degrees of freedom, i.e. translation in the x, y and z directions and rotation about the x, y and z axes. Kinematic constraints can be applied to reduce the number of degrees of freedom. These kinematic constraints are due to mating surfaces which are capable of form variations. Thus, form degrees of freedom may be introduced in the constrained axis directions. Therefore, it is important to know the kinematic degrees of freedom as well as form degrees of freedom associated with each joint type.

In the following subsections, each kinematic joint will be illustrated, indicating 1) the kinematic degrees of freedom, 2) form variation degrees of freedom and 3) the rotational and translational operators required in the engineering model of an assembly. In figure 3.2 through 3.13 the degrees of freedom are indicated on each joint axis and a table of form variation operators. The names of the form variations have been abbreviated as follows:

- FLA - Flatness
- STR - Straightness
- CIR - Circularity
- CYL - Cylindricity
- PRO - Profile
- PAR - Parallelism
- PER - Perpendicularity
- ANG - Angularity
RUNOUT - Runout

CON - Concentricity

POS - Position

3.2.1 Planar

The planar joint allows for the case where two planar surfaces are in contact with each other. Figure 3.2 illustrates the type of contact this joint type represents as well as the local axes orientation.

![Diagram of planar joint with DOF's](image)

**Figure 3.2 Planar Joint with DOF's**

The planar joint introduces a total of six degrees of freedom, three kinematic (K) and three form (F). The three kinematic degrees of freedom come from the possibility of actual physical movement along the x and z axes, and a rotation about the y axis. The form degrees of freedom adds rotation variations about the x and z axes (Rx Rz),
and a translation variation along the y axis (Ty). The applicable form tolerances responsible for the variations are shown in the table at the bottom of figure 3.2.

The translation in the y direction is added to the **RUN OUT** form tolerance because of the variation possible in that direction. It can be more easily seen if the plane is thought of as a rotating disk (e.g. hard disk platter). The variation in the y direction is not included in the other form tolerances since the translation variation in that direction can be absorbed into the size tolerance.

3.2.2 Rigid

The rigid joint is a special case of the planar joint where there are no kinematic degrees of freedom. Figure 3.3 illustrates the type of contact this joint type represents as well as the local axes orientation.

![Figure 3.3 Rigid Joint with DOF's](image)

The rigid joint has no kinematic (K) degrees of freedom due to the fact that this joint is viewed as being rigid such as a weld joint or a joint constrained in such a way as
to prohibit all kinematic motion. It does, however, introduce five form (F) degrees of freedom. The form degrees of freedom are rotations about the x and z axes (Rx Rz) and translations along the x, y and z axes (Tx Ty Tz). The applicable form tolerances responsible for the variations are shown in the table at the bottom of figure 3.3.

The translation along the y axis is, as in the planar joint, introduced by the RUN OUT form tolerance only (see section 3.2.1). The y axis translation for the remainder of the form tolerances is incorporated into the part size tolerance.

3.2.3. Cylindrical Slider

The cylindrical slider joint is the first of four "slider" type joints which include a planar surface. Figure 3.4 illustrates the type of contact this joint type represents as well as the local axes orientation.

![Diagram of Cylindrical Slider Joint]

Figure 3.4 Cylindrical Slider Joint with DOF's
The cylindrical slider joint allows for four kinematic (K) degrees of freedom. These consist of rotations about the y and z axes (Ry Rz) and translations along the x and z axes (Tx Tz). The form degrees of freedom consist of a rotation about the x axis (Rx) and a translation along the y axis (Ty).

This particular "slider" type of joint introduces two sets of variations. One pertaining to the cylinder and the other associated with the plane. As seen in the table in figure 3.4 the cylindrical (C) side of the joint has a different set of variations than does the planar (P) side of the joint. For example, circularity and cylindricity apply to the cylinder but not the plane.

3.2.4 Edge Slider

The edge slider joint is the second of four "slider" joints and is a special case of the cylindrical slider described above, in which the radius of the contact surface is reduced to zero. This joint type allows for a line contact on a planar surface. Figure 3.5 illustrates the type of contact this joint type represents as well as the local axes orientation.
Figure 3.5 Edge Slider Joint with DOF's

The edge slider joint is similar to the cylindrical slider. It provides for the same four kinematic and two form degrees of freedom, but the form tolerances and variations associated with the joint differs. The four kinematic (K) degrees of freedom are rotations about the y and z axes (Ry Rz) and translations along the x and z axes (Tx Tz). The form degrees of freedom (F) are a rotation about the x axis (Rx) and a translation along the y axis (Ty).

The edge (E) side of the joint is very similar to the cylindrical slider's cylinder side (see figure 3.4) except the circularity, cylindricity and run out form tolerances do not apply. On the plane (P) side, the edge slider gives a translation along the y axis (Ty) in addition to the rotation about the x axis (Rx) for flatness and straightness and a translation along the y axis (Ty) and rotation about the x axis (Rx) for runout.
3.2.5 Spherical Slider

The spherical slider joint is third in the list of "slider" joints. It allows for any spherical type surface in contact with a planar surface. Figure 3.6 illustrates the type of contact this joint type represents as well as the local axes orientation.

![Spherical Slider Joint with DOF's](image)

The spherical slider joint is a specialized joint that has five kinematic (K) degrees of freedom but only one form (F) degree of freedom. The kinematic degrees of freedom are rotations about the x, y and z axes (Rx Ry Rz) and translations along the x and z axes (Tx Tz). The lone form degree of freedom in a translation along the y axis (Ty). There are very few form tolerances that give rise to form variations for this joint type.
3.2.6 Point Slider

The point slider joint is the last of the four "slider" joints. This joint allows for a point contact with a planar surface. Figure 3.7 illustrates the type of contact this joint type represents as well as the local axes orientation.

![Diagram of Point Slider Joint with DOF's](image)

<table>
<thead>
<tr>
<th>PT</th>
<th>FLA</th>
<th>STR</th>
<th>CIR</th>
<th>CYL</th>
<th>PRO</th>
<th>PAR</th>
<th>PER</th>
<th>ANG</th>
<th>RUN-OUT</th>
<th>CON</th>
<th>POS</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Ty</td>
</tr>
</tbody>
</table>

Figure 3.7 Point Slider Joint with DOF's

The point slider joint is a special case of the spherical slider joint where the point of contact can be looked at as a sphere with zero radius. Consequently, as with the spherical slider joint, there are five kinematic (K) degrees of freedom and one form (F) degree of freedom.

The only form tolerances that contribute to form variations with this joint are run out and position. Position is associated with the point side of the joint giving a
translation along the y axis (Ty), whereas, run out introduces another translation along the y axis (Ty) associated with the plane side of the joint.

3.2.7 Cylindrical

The cylindrical joint is formed by two cylindrical surfaces, one inside the other. It is important to note that clearance between the two cylinders is not considered. Figure 3.8 illustrates the type of contact this joint type represents as well as the local axes orientation.

![Cylindrical Slider Joint with DOF's](image)

<table>
<thead>
<tr>
<th>FLA</th>
<th>STR</th>
<th>CIR</th>
<th>CYL</th>
<th>PRO</th>
<th>PAR</th>
<th>PER</th>
<th>ANG</th>
<th>RUN-OUT</th>
<th>CON</th>
<th>POS</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>Rx Rz</td>
<td>Rx Rz</td>
<td>Rx Rz</td>
<td>Rx Rz</td>
<td>Rx Rz</td>
<td>Rx Rz</td>
<td>Rx Rz</td>
<td>Rx Rz</td>
<td>Tx Tz</td>
<td>Rx Rz</td>
</tr>
</tbody>
</table>

Figure 3.8 Cylindrical Slider Joint with DOF’s

The cylindrical joint allows for two kinematic (K) degrees of freedom and four form (F) degrees of freedom. The kinematic degrees of freedom are rotation about the y axis (Ry) and translation along the y axis (Ty). The form degrees of freedom take up variations along and about the other two axes (i.e. Rx Tx and Rz Tz).

Flatness does not apply in this case since both surfaces are cylindrical. Straightness, Parallelism, Perpendicularity and Angularity which describe a cylindrical
tolerance zone about the cylinder axis. The axis of either cylinder is permitted to tilt within the limits of the tolerance zone, giving rise to angular variations Rx or Rz.

It is also interesting to note that concentricity and position are the only form tolerances that contribute translation variations. Since concentricity and position are location feature controls, they should be specified if translational variations are permitted.

3.2.8 Revolute

The revolute joint could be viewed as a special case of the cylindrical joint in that the kinematic degree of freedom along the y axis in the cylindrical joint is not found in the revolute joint. This joint can also be thought of as a "pin" joint. Figure 3.9 illustrates the type of contact this joint type represents as well as the local axes orientation.

![Figure 3.9 Revolute Joint with DOF's](image)

The revolute joint (or pin joint) allows for one kinematic (K) degree of freedom and four form (F) degrees of freedom. The single kinematic degree of freedom introduces a rotation variation about the y axis (Ry). The form degrees of freedom
contribute rotation variations about the x and z axes (Rx Rz) and translation variations along the x and z axes (Tx Tz) also.

Again, as with the cylindrical joint, flatness does not apply since the two contact surfaces are cylindrical and concentricity and position are the only form tolerances that contribute any translational variations. It is important to note that there is no translation variation, either kinematic or form, along the y axis for this joint.

3.2.9 Prismatic

The prismatic joint allows for three planar surfaces of each part to be in contact in a channel like arrangement. It is important to note that clearance between the mating surfaces is not considered. Figure 3.10 illustrates the type of contact this joint type represents as well as the local axes orientation.

<table>
<thead>
<tr>
<th>FLA</th>
<th>STR</th>
<th>CIR</th>
<th>CYL</th>
<th>PRO</th>
<th>PAR</th>
<th>PER</th>
<th>ANG</th>
<th>RUN-OUT</th>
<th>CON</th>
<th>POS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rz Ry Rz</td>
<td>Rz Ry Rz</td>
<td>N/A</td>
<td>N/A</td>
<td>Rz Ry Rz</td>
<td>Rz Ry Rz</td>
<td>Rz Ry Rz</td>
<td>Rz Ry Rz</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Figure 3.10 Prismatic Joint with DOF's
The prismatic joint contributes one kinematic (K) degree of freedom and three form (F) degrees of freedom. The one kinematic degree of freedom comes from a translation variation along the z axis (Tz). The three form degrees of freedom are all rotations about each axis (Rx Ry Rz) due to surface waviness.

Circularity, cylindricity, run out, concentricity and position do not apply due to the nature of this joint as can be clearly seen in figure 3.10. The translation variations that would seem to be obviously present can be accounted for in the size tolerances associated with the applicable part features.

3.2.10 Crossed Cylinders

The crossed cylinders joint type allows for the contact of any cylindrical surface in contact with any other cylindrical surface. Figure 3.11 illustrates the type of contact this joint type represents as well as the local axes orientation.

<table>
<thead>
<tr>
<th>FLA</th>
<th>STR</th>
<th>CIR</th>
<th>CYL</th>
<th>PRO</th>
<th>PAR</th>
<th>PER</th>
<th>ANG</th>
<th>RUN-OUT</th>
<th>CON</th>
<th>POS</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Ty</td>
<td>N/A</td>
<td>Ty</td>
</tr>
</tbody>
</table>

Figure 3.11 Crossed Cylinders Joint with DOF's
The crossed cylinders joint allows for five kinematic (K) degrees of freedom and one form (F) degree of freedom. The five kinematic degrees of freedom consist of rotations about all three axes (Rx Ry Rz) and translations along the x and z axes (Tx Tz). The lone form degree of freedom contributes a translational variation along the y axis (Ty).

Due to the nature of this joint, the form tolerances introduces a translational variation along the y axis (Ty) only. Any remaining variations can be absorbed by the size tolerances.

3.2.11 Parallel Cylinders

The parallel cylinder joint type is a special case of the crossed cylinders joint type described above. This is a crossed cylinders joint with the axes of the two cylinders positioned parallel to each other. Figure 3.12 illustrates the type of contact this joint type represents as well as the local axes orientation. The two cylinders may make contact externally or one cylinder may be inside the other making internal contact as shown below. The local axes and degrees of freedom for both internal or external are the same.
Figure 3.12 Parallel Cylinders Joint with DOF's

The parallel cylinders joint contributes two kinematic (K) degrees of freedom and two form (F) degrees of freedom. The two kinematic degrees of freedom are a rotation about the z axis (Rz) and a translation along the z axis (Tz). The two form degrees of freedom are a rotation about the x axis (Rx) and a translation along the y axis (Ty).

Flatness and concentricity form tolerances do not apply for this particular joint type. Unlike the crossed cylinders joint, the parallel cylinders has a line contact that introduces a rotation about the x axis (Rx) and also sees a rotation contribution from the parallelism, perpendicularity and angularity form tolerances as well.

3.2.12 Spherical

The spherical joint allows for two spherical surfaces in contact with each other. This joint can also be thought of as a "ball" joint. Figure 3.13 illustrates the type of contact this joint type represents as well as the local axes orientation.
The spherical joint is an interesting joint due to its nature. There are, in actuality, three kinematic (K) degrees of freedom and three form (F) degrees of freedom. However, one kinematic degree of freedom (a rotation about the y axis) is redundant since it is a rotation about the input shaft (i.e. the shaft at the bottom of the joint in figure 3.13), there are no variations contributed by this Ry degree of freedom. It is shown only to clarify the problem. The three form degrees of freedom are all translations along each axis (Tx Ty Tz).

The variations associated with this joint can be viewed as a spherical tolerance zone located at the origin of the local axes where that origin may vary anywhere inside that sphere. Thus, translation variations are the only applicable form tolerances.
3.3 Form Variation Model Summary

The following table summarizes the form variation models defined for each of the twelve kinematic joint types.

Table 3.1 Rotational and Translational Variations Associated with the Corresponding Geometric Form Tolerance-Kinematic Joint Combination

<table>
<thead>
<tr>
<th>JOINTS</th>
<th>FLA</th>
<th>STR</th>
<th>CIR</th>
<th>CYL</th>
<th>PRO</th>
<th>PAR</th>
<th>PER</th>
<th>ANG</th>
<th>RUN-OUT</th>
<th>CON</th>
<th>POS</th>
</tr>
</thead>
<tbody>
<tr>
<td>RIGID</td>
<td>Rx</td>
<td>Rx</td>
<td>N/A</td>
<td>N/A</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
</tr>
<tr>
<td>REVOLUTE</td>
<td>N/A</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
</tr>
<tr>
<td>CYLINDRICAL</td>
<td>N/A</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
</tr>
<tr>
<td>PRISMATIC</td>
<td>Rx</td>
<td>Rx</td>
<td>N/A</td>
<td>N/A</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
</tr>
<tr>
<td>PLANAR</td>
<td>Rx</td>
<td>Rx</td>
<td>N/A</td>
<td>N/A</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
<td>Rx</td>
</tr>
<tr>
<td>EDGE SLIDER</td>
<td>N/A</td>
<td>Ty</td>
<td>Ty</td>
<td>N/A</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>CYLINDRICAL SLIDER</td>
<td>N/A</td>
<td>Ty</td>
<td>Ty</td>
<td>N/A</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>POINT SLIDER</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>SPHERICAL SLIDER</td>
<td>N/A</td>
<td>Ty</td>
<td>N/A</td>
<td>N/A</td>
<td>Ty</td>
<td>N/A</td>
<td>N/A</td>
<td>Ty</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>CROSSED CYLINDERS</td>
<td>N/A</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>N/A</td>
<td>Ty</td>
<td>N/A</td>
<td>Ty</td>
</tr>
<tr>
<td>SPHERICAL</td>
<td>N/A</td>
<td>N/A</td>
<td>Tx</td>
<td>Tx</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>N/A</td>
<td>Ty</td>
<td>N/A</td>
<td>Ty</td>
</tr>
<tr>
<td>PARALLEL CYLINDERS</td>
<td>N/A</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>Ty</td>
<td>R</td>
<td>R</td>
<td>Ty</td>
<td>R</td>
<td>Ty</td>
</tr>
</tbody>
</table>

** R = Rotational variation,  T = Translational variation

It is important to note that the x, y, z axis for each of these joints is a local axis specific to each joint. The form tolerances that do not apply to a particular joint are designated as N/A (not applicable). The kinematic joint types and their respective local axes will be looked at more closely in the following sections.
It should be mentioned that run out is not a singularly unique "form" tolerance but, is a composite of form, orientation and location effects. But it was treated along with the form variations in this development for completeness.

3.4 Applying Form Tolerances in Practice

Sometimes it is not clear whether or not to apply form tolerances. For example, if the size of a part is sampled by a Coordinate Measuring Machine (CMM), then the resulting distribution includes local waviness as well as size. However, if size is measured with a micrometer or some other device with flat anvil surfaces which are unable to detect waviness, then waviness could result in independent variations. But, then again it depends on what kind of mating surfaces are brought in contact. If the period of waviness is much shorter than the length of contact between mating surfaces, then the waviness can not result in rotational variation. Because of the possibility of variations being introduced by the use of these different methods, care must be exercised to provide for them appropriately in the tolerance analysis.

For instance, flatness, straightness, profile, parallelism, perpendicularity and angularity tolerances, in some instances can be neglected since the ANSI Y14.5M-82 standard states that these variations cannot exceed the size tolerance limits. That is to say that any size variation plus any flatness variation, for example, cannot be greater than the size tolerance specified. It might be argued that the flatness variation does not introduce an independent variation into an assembly and that it is accounted for by the size variation. But, this really depends on how the feature size is measured.

3.5 Conclusion

There are other joints in 3-D kinematics such as universal, gear, helical, etc. which are very useful for kinematic mechanisms. However, they have very little if any use for assembly modeling. If these particular joint types need to be included, they can be modeled using one or a combination of the joint types discussed above. For example, a very basic universal joint may be modeled using two revolute joints.

Many different applications have been examined to include all joint and form tolerance possibilities presented in this section. However, there may be some applications that would add more combinations than have been considered. These "new" possibilities may be handled by substituting an existing joint type (as you would
for the universal, etc joint types mentioned in the paragraph above) that would give the appropriate variations required or by using a zero-length vector positioned such that the needed variation will be taken into account.
Chapter 4

STATISTICAL ANALYSIS WITH FORM TOLERANCES

Detailed theory of how to include form variations in a vector loop model will be presented in this chapter. Special attention will be given to determining the sensitivities for use in the linearized equations used in analyzing assemblies for dimensional and feature variations.

First, a 2-D system will be examined to illustrate the concepts, then a 3-D system will be presented to show how the concepts may be extended to complex assemblies.

4.1 Open and Closed Loop Equations

As shown in chapter 2, linearized loop equations may be expresses as follows:

\[ du = -[B^{-1}A]dx - [B^{-1}F]d\alpha \quad \text{(closed loop)} \]
\[ dh = [C - DB^{-1}A]dx + [G - DB^{-1}F]d\alpha \quad \text{(open loop)} \]

\( du \) and \( dh \) represent resultant assembly variations which are computed in terms of the component variations \( dx \) and \( d\alpha \). These terms are then used to form the Worst-Case and Statistical Root-Sum-Squared expressions as discussed previously in chapter 2.

The key to obtaining the above mentioned estimates of assembly variations is the determination of the \( A, B, C \) and \( D \) matrices, which consist of the dimensional and kinematic variations, and the \( F \) and \( G \) matrices containing the form variations in the assembly. The original loop equations are given as:

\[ dh = Adx + Bdu + Fd\alpha = 0 \quad \text{for closed loops and}, \]
\[ dh = Cdx + Ddu + Gd\alpha \quad \text{for open loops}. \]

Expanding, the linearized closed loop equations may be written as:

\[ dh_x(x_i, u_j, \alpha_k) = \sum \left( \frac{\partial h_x}{\partial x_i} dx_i \right) + \sum \left( \frac{\partial h_x}{\partial u_j} du_j \right) + \sum \left( \frac{\partial h_x}{\partial \alpha_k} d\alpha_k \right) = 0 \]
\[ dh_y(x_i, u_j, \alpha_k) = \sum \left( \frac{\partial h_y}{\partial x_i} dx_i \right) + \sum \left( \frac{\partial h_y}{\partial u_j} du_j \right) + \sum \left( \frac{\partial h_y}{\partial \alpha_k} d\alpha_k \right) = 0 \]

\[ dh_\theta(x_i, u_j, \alpha_k) = \sum \left( \frac{\partial h_\theta}{\partial x_i} dx_i \right) + \sum \left( \frac{\partial h_\theta}{\partial u_j} du_j \right) + \sum \left( \frac{\partial h_\theta}{\partial \alpha_k} d\alpha_k \right) = 0 \]

where \( dx_i \) are the specified tolerances on the independent dimensions and \( du_j \) are the resultant variations in the dependent assembly dimensions. The equations have been expanded to include form and feature tolerances \( d\alpha_k \).

Our attention will be mainly focused on the \( F \) and \( G \) matrices and how the partial derivatives may be obtained and evaluated.

### 4.2 2-D Example

The Tape Hub Lock assembly shown in figure 4.1 contains both open and closed loops. Because the loops are all confined to a single plane this is a 2-dimensional problem which will be used to illustrate the concepts presented. The 2-dimensional environment is much easier to present and, once the 2-D concepts are understood, the jump to 3-D is made easier.
For the locking tape hub problem, the single closed loop shown in Figure 4.1 gives the following set of nonlinear equations for the dimension vectors:

\[ h_x = b \cos(90) + a \cos(0) + u \cos(\theta) + r \cos(\theta - 90) + (e + i) \cos(\theta + \Phi - 90) + g \cos(\theta + \Phi - 180) + R_L \cos(\theta + \Phi - 270) + h \cos(\theta + \Phi - 180) = 0 \]

\[ h_y = b \sin(90) + a \sin(0) + u \sin(\theta) + r \sin(\theta - 90) + (e + i) \sin(\theta + \Phi - 90) + g \sin(\theta + \Phi - 180) + R_L \sin(\theta + \Phi - 270) + h \sin(\theta + \Phi - 180) = 0 \]

\[ h_\theta = 90 - 90 + \theta - 90 + \Phi + 0 - 90 - 90 + 90 + 90 = 0 \]

where \( a, b, e, g, h, i, \) and \( \theta \) are the independent manufactured dimensions and \( R_L, u \) and \( \Phi \) are the dependent kinematic dimensions.

Adding form variations to the equations shown above is not a difficult process but does require understanding of the type of variations introduced by each form tolerance applied and their relationships to the joint axes. Consider the cylindrical slider type contact between the arm and the plunger (at contact point of vectors \( u \) and \( r \)). A "zero length" vector needs to be inserted at the point of contact with direction...
perpendicular to the surface. This "zero length" vector has a form variation \( \alpha_1 \) with \( \pm d\alpha \) tolerance. The contributions to the resulting closed loop equations by the form variations are given as follows:

\[
\begin{align*}
h_x &= \text{(dimension vectors)} + \alpha_1 \cos(\theta - 90) \\
h_y &= \text{(dimension vectors)} + \alpha_1 \sin(\theta - 90) \\
h_\theta &= \text{No contribution by the translational form variations}
\end{align*}
\]

Evaluating partial derivatives gives:

\[
\begin{align*}
\frac{\partial h_x}{\partial \alpha_1} &= \cos(\theta - 90) \\
\frac{\partial h_y}{\partial \alpha_1} &= \sin(\theta - 90)
\end{align*}
\]

Since the nominal value of \( \alpha_1 \) is zero, \( \partial h_x/\partial x_i, \partial h_y/\partial \theta_i \), etc. are not affected because they are evaluated at the nominal value of all variables. Thus, any new terms containing \( \alpha_1 \) will drop out. The only contribution will appear in the F matrix. The A and B matrices are independent of \( \alpha_1 \) variations.

Similarly, at the planar joint (located between the arm and base at the intersection of vectors \( g \) and \( R_L \)) another "zero length" vector \( \alpha_2 \) can be added at the point of contact, acting perpendicular to the surface. Only in this case we get a rotational variation.

\[
h_x = b \cos(90) + a \cos(0) + u \cos(\theta) + r \cos(\theta - 90) + \alpha_1 \cos(\theta - 90) + (e + i) \cos(\theta + \Phi - 90) + g \cos(\theta + \Phi - 180) + \alpha_2 \cos(\theta + \Phi - 180 + \alpha_3) + R_L \cos(\theta + \Phi - 270 + \alpha_3) + h \cos(\theta + \Phi - 180 + \alpha_3) = 0
\]

where \( \alpha_3 \) is an angular variable acting at the point of contact between vectors \( h \) and \( R_L \). The other nonlinear equations are handled similarly. The partial of this equation is,

\[
\frac{\partial h_x}{\partial \alpha_2} = -\alpha_2 \sin(\theta + \Phi - 180 + \alpha_3) - R_L \sin(\theta + \Phi - 270 + \alpha_3) - h \sin(\theta + \Phi - 180 + \alpha_3)
\]

Note that \( d\alpha_2 = 0 \), so the partial \( \partial h_x/\partial \alpha_2 \) did not contribute a translational variation. But the rotation variation propagated through the remaining vectors in the loop.

The rotation equation \( h_\theta \) yields
h_0 = \theta + \Phi - 90 + \alpha_3 = 0 \text{ and } \partial h_0 / \partial \alpha_3 = +1

4.3 Physical Interpretation

Translational form variation:

When evaluating partial derivatives, all variables are held at their nominal values. Therefore, each vector length and direction does not change. This produces a rigid body shift in the the remaining vectors in the loop such that vectors dh and d\alpha are equal in magnitude and direction. Figure 4.2 shows the resultant rigid body shift due to a translational variation located at the cylindrical slider point of contact relating to \alpha_1 mentioned above.

Figure 4.2  Rigid Body Shift Due to Translational Form Variation
The components of the resultant closure error vector are given as follows:

\[ dh_X = dh \cos(\theta - 90) = d\alpha \cos(\theta - 90) \]

\[ dh_Y = dh \sin(\theta - 90) = d\alpha \sin(\theta - 90) \]

and the partial is given as \( \partial h_X / \partial \alpha = \cos(\theta - 90) \) and \( \partial h_Y / \partial \alpha = \sin(\theta - 90) \) and \( \partial h_\theta / \partial \alpha = 0 \) as shown previously.

**Rotational form variation:**

Similarly, since the lengths and relative angles of the vectors are held at their nominal values, only the relative angle \( \alpha_3 \) changes. This produces a rigid body rotation of all remaining vectors in the loop. As figure 4.3 illustrates, this rigid body rotation can be replaced by a single resultant vector \( Z \) rotated through a small angle \( d\alpha_3 \).

![Figure 4.3 Rigid Body Rotation Due to Rotational Form Variation](image-url)
Due to the small angle variation $\alpha_3$, the resultant closure error vector $dh$ can be written as follows:

$$dh = Z \, d\alpha_3 \quad (\alpha_3 \text{ in radians})$$

By the same procedure, the partials can be written as:

$$\partial h_x/\partial \alpha_3 = dh_x/d\alpha_3$$

$$\partial h_y/\partial \alpha_3 = dh_y/d\alpha_3$$

$$\partial h_\theta/\partial \alpha_3 = +1$$

In summary, to introduce form variations into a 2-D vector loop, we merely need to move to the node or joint where the form variation occurs and generate the appropriate translational or rotational variation in the direction of the form degree of freedom for that joint. From the corresponding change at the loop origin (closure error), we may calculate the sensitivity. By repeating this process node-by-node the $F$ matrix is generated.

### 4.4 Extension to 3-D Form Variations

Three dimensional sensitivities are more complex and therefore more difficult to develop. For example, there are six 3-D linear and rotation equations (3 linear and 3 rotational) compared to only three for 2-D. They can be shown more easily by building on the 2-D equations presented in the previous section. Figure 4.4 shows a vector in a 2-D coordinate system.
Figure 4.4  Vector in 2-D Coordinate System

If you solve for the projection of the vector on the X and Y axes in figure 4.4, we arrive with the results in the equations below,

\[ dh_x = dh \cos(\theta), \text{ and } \]
\[ dh_y = dh \sin(\theta). \]

These equations can also be written as:

\[ dh_x = dh \cos(\theta), \text{ and } \]
\[ dh_y = dh \cos(\phi), \]

i.e., the direction cosines for the vector.

This same vector can also be shown in a 3-D coordinate system as shown in figure 4.5.
Similarly, in 3-D we still arrive at the same conclusions as in the 2-D example above using direction cosines, except with the addition of the $\gamma$ direction cosine.

\[
\begin{align*}
\text{dh}_x &= \text{dh} \cos(\theta), \\
\text{dh}_y &= \text{dh} \cos(\phi), \\
\text{dh}_z &= \text{dh} \cos(\gamma).
\end{align*}
\]

Thus, the generalized vector equation in terms of direction cosines is,

\[
\text{dh} = \text{dh} \cos(\theta) + \text{dh} \cos(\phi) + \text{dh} \cos(\gamma)
\]

With the 3-D equations defined, we can proceed to extend the 2-D concept into 3-D. The 3-D and 2-D concepts are actually the same, except that the dh vector is now 3-dimensional as shown in figure 4.5. The translational partials for 3-D are shown below,
\[ \frac{\partial h_x}{\partial \alpha_1} = \cos(\theta) \]
\[ \frac{\partial h_y}{\partial \alpha_1} = \cos(\phi) \]
\[ \frac{\partial h_z}{\partial \alpha_1} = \cos(\gamma) \]

Rotation becomes a rigid body rotation about a skewed axis so you get, in general, three components for closure error vector \( dh \):
\[ \frac{\partial h_x}{\partial \alpha_3} \approx \frac{dh_x}{d\alpha_3} \]
\[ \frac{\partial h_y}{\partial \alpha_3} \approx \frac{dh_y}{d\alpha_3} \]
\[ \frac{\partial h_z}{\partial \alpha_3} \approx \frac{dh_z}{d\alpha_3} \]

The three rotation equations are much more difficult to develop and I will not expend a lot of effort in doing so. The derivation of 3-dimensional translation and rotation equations is discussed more fully by Jinsong Gao [Gao, 1992].

The partial derivatives of the three rotation equations may now be expressed. Translational variations do not contribute since no lengths appear in the rotation equations.
\[ \frac{\partial h_{\theta x}}{\partial \alpha} = 0 \]
\[ \frac{\partial h_{\theta y}}{\partial \alpha} = 0 \]
\[ \frac{\partial h_{\theta z}}{\partial \alpha} = 0 \]

Consider a unit rotation vector about a skewed axis, as shown in figure 4.6. The sensitivities have been found to be equal to the direction cosines of the rotation vector,
\[ \frac{\partial h_{\theta x}}{\partial \alpha_3} = \cos(\beta_x) \]
\[ \frac{\partial h_{\theta y}}{\partial \alpha_3} = \cos(\beta_y) \]
\[ \frac{\partial h_{\theta z}}{\partial \alpha_3} = \cos(\beta_z) \]
Figure 4.6  Unit Rotation Vector About Skewed Axis

The next chapter will present detailed numerical examples.

4.5 Distributions of Form Tolerances

The statistical analysis performed in the above methods are based on normal distributions such as the 3-σ normal distribution illustrated in figure 4.7.
Figure 4.7  Normal Distribution Curve

This is a simple distribution easily incorporated in computer aided analysis. It is also a fairly simple process to convert to a normal distribution from a uniform distribution, and one sided distributions tend towards normal distributions if there are enough variations present in the assembly.

The justification for using normal distribution rather than other more sophisticated distributions is given as follows:

- CATS (Computer Aided Tolerance Selection) is a design tool not a manufacturing tool. Designers estimate effects of manufacturing variations long before any parts are actually made. Since the parts have not been manufactured yet, there is no real data available on the specific parts to perform more complicated statistical analysis on. Therefore, at this design stage, you can only estimate, at best, from the history of the manufacturing processes.

- We don't know what distributions really are. Distributions haven't been characterized by anyone and so there are no specific guidelines to tell us which type of distribution should be used.

- Form tolerances are generally not major contributors to the overall assembly variation. The log-normal distribution may be a better approximation, but the
distinction between it and normal distribution is not that significant when the form tolerance contribution is minor.

- It has been well established that if there are more than five components in an assembly, which most assemblies have, the resultant distribution tends to be normal even when the component tolerances deviate from normal.

Because of the above mentioned reasons, it has been determined that using a normal distribution is well justified.

4.6 Envelope Rule

Under some conditions, summing form tolerances with dimensional tolerances may violate Rule No. 1 of the ANSI Y14.5 standard. Rule No. 1 is known as the "Envelope Principle", which states that form variations may not cause a part to exceed the envelope of the perfect form at Maximum Material Condition (MMC).

This rule can only be violated, for example, by a worst case analysis in which the size variation and a form variation of a component act in the same direction.

Goodrich has discussed this possibility at length [Goodrich, 1991]. He proposed possible approaches to prevent Rule No. 1 from being violated. The simplest solution is to let the offending dox go to zero for worst case analysis.

For statistical analysis, Rule No. 1 does not apply. Variations are summed statistically and measured from the nominal or center of the tolerance band. Violation of Rule No. 1 is always possible according to some small probability.
Chapter 5

CASE STUDIES

Presented in this chapter are four sample problems involving varying combinations of 3-dimensional joint-types and feature controls that were analyzed using 3-D methods for tolerance analysis. Each example will be shown first, considering only the size variations (which assumes that all parts are at a perfect form or shape), and secondly, introducing form variations to the assembly for analysis. The purpose of these tolerance analyses is to predict the resultant variation on assembly dimensions.

5.1 High-speed Magnetic Tape Drive Hub Lock

This tape hub lock assembly is used on high-speed mass storage units connected to mainframe computer systems. Figure 5.1 shows an exploded view of the tape hub lock assembly.

![Hub Lock Assembly Diagram]

Figure 5.1 Exploded View of Tape Hub Lock
(Courtesy of Storage Technology Corp.)
As can be seen from the figure shown above, the tape hub lock is a fairly complicated assembly even though the overall size is not that big. This problem will be analyzed in the locked position (i.e. with the tape mounted). The three arms of the hub make contact with the tape in the locked position when the yoke is forced down when the latch is locked. This pushes the arms outward to make contact with the tape. This brings up the possibility of the yoke making contact with the hub before the arms make contact with the tape. Therefore, enough clearance must be left between the yoke and the hub to prevent this from occurring.

Figure 5.2 shows a simplified drawing of the hub lock assembly. This drawing is sufficient to show individual parts and the vector loop paths needed for the analysis.

![Figure 5.2 Simplified Tape Hub Lock Assembly](image)

Figure 5.3 shows the vector loops required for the analysis of this problem and the form tolerances applied.
There are five loops in all. The number of loops needed in an assembly is calculated from equation 5.1 [Robison, 1989].

\[
\text{#loops} = \text{#joints} - \text{#parts} + 1
\]  

(5.1)

This equation, for this problem, turns out to be 10 joints - 6 parts +1 = 5 loops. Three loops are used to tie the dimensions of the hub, yoke and arms together and two more loops tie in the hub and tape reel. Figures 5.4 and 5.5 show the individual vector loops defined for this problem. Notice that the three loops in each of the arms are identical when overlayed on each other, this allows the three loops to be shown as one in figure 5.4.
This problem might be viewed as a set of 2-dimensional problems since each loop is defined by a single plane. However, variations in one plane cause variations out of that plane so this is a 3-dimensional problem and will be handled as such.

The setup for the analysis includes some assumptions made in order to simplify the solution of this problem: 1) the plane of the top of the hub is perpendicular to the plane of the movement of the yoke and the arms, 2) each vector loop is in a single
plane, 3) the yoke moves uniformly up and down the same amount on all three arms, and 4) the arms are all assumed to be 120 degrees apart with any variation of this angle considered negligible to the solution.

The dimensions involved in the vector loops are of two types: 1) independent dimensions which are specified or manufactured dimensions and 2) dependent dimensions which are the assembly dimensions of how the assembly should fit together. The independent dimensions are listed in table 5.1 along with their drawing designators, brief descriptions and associated tolerances. The dependent dimensions are listed likewise in table 5.2.

Table 5.1 Independent Dimensions for Hub Lock Problem

<table>
<thead>
<tr>
<th>Vector</th>
<th>Description</th>
<th>Dimension</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Height of datum to bottom of yoke</td>
<td>0.398</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(same for loops 1-3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>Radius of yoke to bottom edge</td>
<td>1.355</td>
<td>0.002</td>
</tr>
<tr>
<td>θ</td>
<td>Angle of yoke cam to bottom edge</td>
<td>75°</td>
<td>0.5°</td>
</tr>
<tr>
<td>r</td>
<td>Corner radius at contact surface of arm</td>
<td>0.06</td>
<td>0.002</td>
</tr>
<tr>
<td>e</td>
<td>Width of arm minus corner radius</td>
<td>0.309</td>
<td>0.003</td>
</tr>
<tr>
<td>i</td>
<td>Thickness of rubber pad on arm</td>
<td>0.05</td>
<td>0.002</td>
</tr>
<tr>
<td>g</td>
<td>Height of arm minus corner radius</td>
<td>0.488</td>
<td>0.004</td>
</tr>
<tr>
<td>Rt1, Rt2, Rt3</td>
<td>Tape inner radius for loops 4-5</td>
<td>1.8546</td>
<td>0.004</td>
</tr>
</tbody>
</table>
Table 5.2  Dependent Dimensions for Hub Lock Problem

<table>
<thead>
<tr>
<th>Vector</th>
<th>Description</th>
<th>Dimension</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1,R2,R3</td>
<td>Hub assembly outer radius for loops 1-3 (Rh1,Rh2,Rh3 for loops 4-5)</td>
<td>1.8546</td>
<td>?</td>
</tr>
<tr>
<td>U1,U2,U3</td>
<td>Location of contact point of arm on yoke cam for loops 1-3</td>
<td>0.3194</td>
<td>?</td>
</tr>
<tr>
<td>Φ1,Φ2,Φ3</td>
<td>Angle between contact radius and horizontal for loops 1-3</td>
<td>15°</td>
<td>?</td>
</tr>
<tr>
<td>Φ4,Φ5</td>
<td>Angle between Rh1 and Rh2</td>
<td>120°</td>
<td>?</td>
</tr>
</tbody>
</table>

Table 5.2 lists fifteen unknown values which will need to be solved for. Three-dimensional assemblies give rise to the possibility of six degrees of freedom at each node analyzed. These are translation in the local X, Y and Z directions and rotation about the local X, Y and Z axes. As can be imagined this process can become a very involved and a laborious process. For this reason, the use of a computer and a analysis program should be used.

3-D Solutions With and Without Form Tolerances

The sensitivities for all dimensions and the values for the fifteen unknown tolerances were found by using a computer based spreadsheet with matrix capabilities. The matrix loop equations were solved using the same spreadsheet.

The calculated tolerances for the dependent variables are given in table 5.4. The solutions were arrived at using two methods: 1) statistical, using the equations found in section 4.2.2 and 2) worst case, using the equations found in sections 4.2.1.

These values show how the assembly will fit together using the assigned tolerances on the independent (manufactured) dimensions and mating part types involved. It is important to remember here that form tolerances have not been taken into account yet in this analysis.
The same vector loop setup will be used for the analysis including form tolerances as was used in the analysis without form tolerances. Table 5.3 lists the form tolerances applied and where they were used along with their tolerance values. Remember that some feature controls have multiple degrees of freedom, some translational and some rotational (refer to chapter 3). There must be a tolerance given for each of those degrees of freedom.

Table 5.3  Form Tolerances for Hub Lock Problem

<table>
<thead>
<tr>
<th>Form Tolerance (Joint Type used)</th>
<th>Feature</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(Tran)</td>
</tr>
<tr>
<td>ANG (Cyl Slider)</td>
<td>Angle at base of yoke and yoke cam for loops 1-3</td>
<td>0.002</td>
</tr>
<tr>
<td>PER (Planar)</td>
<td>Edge of arm in contact with tape for loops 1-3</td>
<td>N/A</td>
</tr>
<tr>
<td>CIR (Cyl Slider)</td>
<td>The inside cylinder of the tape and hub for loops 4 and 5</td>
<td>0.004</td>
</tr>
</tbody>
</table>

The results from both of the spreadsheet calculations are given in table 5.4 below. Both statistical and worst-case results are shown.

Table 5.4  Comparison of Resultant Tolerances

<table>
<thead>
<tr>
<th>dU</th>
<th>Stat w/o</th>
<th>Stat w/</th>
<th>WC w/o</th>
<th>WC w/</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>0.00725</td>
<td>0.00732</td>
<td>0.01383</td>
<td>0.01484</td>
</tr>
<tr>
<td>R1</td>
<td>0.00535</td>
<td>0.00546</td>
<td>0.01427</td>
<td>0.01533</td>
</tr>
<tr>
<td>φ1</td>
<td>0.37698</td>
<td>0.37721</td>
<td>0.34378</td>
<td>0.49905</td>
</tr>
<tr>
<td>U2</td>
<td>0.00725</td>
<td>0.00732</td>
<td>0.01383</td>
<td>0.01484</td>
</tr>
<tr>
<td>R2</td>
<td>0.00535</td>
<td>0.00546</td>
<td>0.01427</td>
<td>0.01533</td>
</tr>
<tr>
<td>φ2</td>
<td>0.37698</td>
<td>0.37721</td>
<td>0.34378</td>
<td>0.49905</td>
</tr>
<tr>
<td>U3</td>
<td>0.00725</td>
<td>0.00732</td>
<td>0.01383</td>
<td>0.01484</td>
</tr>
<tr>
<td>R3</td>
<td>0.00535</td>
<td>0.00546</td>
<td>0.01427</td>
<td>0.01533</td>
</tr>
</tbody>
</table>
When compared to the results from the analysis without form tolerances, it can be seen in table 5.4 that form tolerances can make a difference in the final assembly and should not be neglected.

5.2 Spatial Slider-Crank Mechanism

The slider-crank mechanism shows true 3-dimensional characteristics in its motion. As the crank rotates in one plane the slider translates in an orthogonal plane as shown in Figure 5.6.
Figure 5.6  Spatial Slider-Crank Mechanism

As the crank rotates, the crank lever carries the joint arm around a circular path forcing the slider to slide back and forth with each revolution of the crank lever. The "ball joints" allow the joint arm to pivot as it moves in its path of motion.

The vector loops required for this problem is 1 as calculated from equation 5.1 (4 joints - 4 parts + 1 = 1). Figure 5.7 details the vector loop for this problem and the associated form tolerances. An assumption that the crank lever is at a 45 degree angle off of vertical for this analysis is made.
Figure 5.7 Vector Loop Diagram for Slider-Crank Mechanism

Again the dimensions involved with this loop are the independent (manufactured) and the dependent (assembly) dimensions. The independent and dependent dimensions are listed along with their drawing designators, brief descriptions and associated tolerances in tables 5.5 and 5.6 respectively.
### Table 5.5  Independent Dimensions for Slider-Crank Problem

<table>
<thead>
<tr>
<th>Vector</th>
<th>Description</th>
<th>Dimension</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Distance of crank from base</td>
<td>20.0</td>
<td>0.025</td>
</tr>
<tr>
<td>b</td>
<td>Distance to crank lever line of action</td>
<td>12.0</td>
<td>0.0125</td>
</tr>
<tr>
<td>c</td>
<td>Length of crank lever arm</td>
<td>15.0</td>
<td>0.0125</td>
</tr>
<tr>
<td>d</td>
<td>Length of connecting rod</td>
<td>30.0</td>
<td>0.03</td>
</tr>
<tr>
<td>e</td>
<td>Distance from rod to slider datum</td>
<td>5.0</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

### Table 5.6  Dependent Dimensions for Slider-Crank Problem

<table>
<thead>
<tr>
<th>Vector</th>
<th>Description</th>
<th>Dimension</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>Distance from slider datum to wall</td>
<td>39.7</td>
<td>?</td>
</tr>
<tr>
<td>φ1</td>
<td>Angle variation about arbitrary axis at first ball joint</td>
<td>0°</td>
<td>?</td>
</tr>
<tr>
<td>φ2</td>
<td>2nd angle variation about arbitrary axis at first ball joint</td>
<td>0°</td>
<td>?</td>
</tr>
<tr>
<td>φ3</td>
<td>Angle variation about arbitrary axis at 2nd ball joint</td>
<td>0°</td>
<td>?</td>
</tr>
<tr>
<td>φ4</td>
<td>2nd angle variation about arbitrary axis at 2nd ball joint</td>
<td>0°</td>
<td>?</td>
</tr>
</tbody>
</table>

Table 5.6 lists the unknown variables that will need to be solved for. The spreadsheet program used for the first example will again be used for this analysis.
3-D Solutions With and Without Form Tolerances

As was done in the first example, the unknown tolerances will be calculated first without form tolerances and then again with form tolerances. The calculated tolerances for the dependent variables without form tolerances are given in table 5.8. Again, the solutions are of the statistical and worst case forms used in the previous problem.

Form tolerances will now be added to the analysis and the resultant tolerances calculated. These form tolerances are listed in table 5.7 below.

Table 5.7 Form Tolerances for Slider-Crank Problem

<table>
<thead>
<tr>
<th>Form Tolerance (Joint Type used)</th>
<th>Feature</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CYL (Revolute)</td>
<td>At crank attachment to support</td>
<td>N/A 0.014</td>
</tr>
<tr>
<td>PER (Prismatic)</td>
<td>Contact of slider and channel</td>
<td>N/A 0.036</td>
</tr>
<tr>
<td>PRO (Spherical)</td>
<td>Profile of spherical ball joints</td>
<td>0.002 N/A</td>
</tr>
</tbody>
</table>

The results of the two analyses are shown in table 5.8. The results again show the effect form tolerances can have on the assembly and thus, it is reemphasized that they should not be neglected.

Table 5.8 Comparison of Resultant Tolerances

<table>
<thead>
<tr>
<th>dU</th>
<th>Stat w/o</th>
<th>Stat w/</th>
<th>WC w/o</th>
<th>WC w/</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>0.03508</td>
<td>0.03566</td>
<td>0.05132</td>
<td>0.06540</td>
</tr>
<tr>
<td>ϕ1</td>
<td>0.05794</td>
<td>0.10266</td>
<td>0.08425</td>
<td>0.11278</td>
</tr>
<tr>
<td>ϕ2</td>
<td>0.02857</td>
<td>0.03249</td>
<td>0.04024</td>
<td>0.06877</td>
</tr>
<tr>
<td>ϕ3</td>
<td>0.05584</td>
<td>0.07370</td>
<td>0.08425</td>
<td>0.14027</td>
</tr>
<tr>
<td>ϕ4</td>
<td>0.02857</td>
<td>0.04618</td>
<td>0.04024</td>
<td>0.08432</td>
</tr>
</tbody>
</table>
5.3 3-D Pawl and Ratchet Problem

The third example problem is a 3-D pawl and ratchet assembly. This was modified from a 2-dimensional assembly problem and altered to make a 3-dimensional problem. The ratchet rotates in a single plane while the pawl assembly rotates in an orthogonal plane thus, providing the spatial construction needed for a 3-D analysis. Figure 5.8 shows the pawl and ratchet assembly.

![3-D Pawl and Ratchet Assembly](image)

Figure 5.8 3-D Pawl and Ratchet Assembly

The number of loops required for this problem can be calculated using equation 5.1, i.e. 5 joints - 4 parts +1 = 2. Therefore, two loops are needed to analyze this
problem. Figure 5.9 illustrates the location of these vector loops along with the applied form tolerances.

Figure 5.9  Vector Loops for Pawl and Ratchet Problem

The independent (or manufactured) dimensions and the dependent (assembly) dimensions are listed in tables 5.9 and 5.10 respectively.
<table>
<thead>
<tr>
<th>Vector</th>
<th>Description</th>
<th>Dimension</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Radius of sphere</td>
<td>0.5925</td>
<td>0.0015</td>
</tr>
<tr>
<td>b</td>
<td>Radius of sphere</td>
<td>0.5925</td>
<td>0.0015</td>
</tr>
<tr>
<td>c</td>
<td>Length of base edge</td>
<td>8.38</td>
<td>0.01</td>
</tr>
<tr>
<td>d</td>
<td>Distance from edge of base to vertical</td>
<td>3.7825</td>
<td>0.005</td>
</tr>
<tr>
<td>e</td>
<td>Height of pawl joint from base</td>
<td>8.9312</td>
<td>0.01</td>
</tr>
<tr>
<td>f</td>
<td>Distance of pawl joint from rear edge</td>
<td>1.25</td>
<td>0.005</td>
</tr>
<tr>
<td>g</td>
<td>Thickness of pawl flange</td>
<td>0.375</td>
<td>0.002</td>
</tr>
<tr>
<td>h</td>
<td>Height from pawl joint to top</td>
<td>2.0</td>
<td>0.004</td>
</tr>
<tr>
<td>k</td>
<td>Distance from pawl joint to front</td>
<td>6.0</td>
<td>0.01</td>
</tr>
<tr>
<td>m</td>
<td>Dist from front of pawl to center of sphere</td>
<td>0.75</td>
<td>0.0025</td>
</tr>
<tr>
<td>n</td>
<td>Dist from edge of flange to center of sphere</td>
<td>1.0</td>
<td>0.005</td>
</tr>
<tr>
<td>p</td>
<td>Dist from top of flange to center of sphere</td>
<td>1.5925</td>
<td>0.005</td>
</tr>
<tr>
<td>r</td>
<td>Dist from edge of ratchet to front edge</td>
<td>0.75</td>
<td>0.0025</td>
</tr>
<tr>
<td>s</td>
<td>Height of ratchet joint from base</td>
<td>6.0</td>
<td>0.01</td>
</tr>
<tr>
<td>t</td>
<td>Distance of center of ratchet to side edge</td>
<td>5.0</td>
<td>0.01</td>
</tr>
<tr>
<td>θ1</td>
<td>Angle of tooth surface on ratchet</td>
<td>81.0°</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Table 5.10 Dependent Dimensions for Pawl Problem

<table>
<thead>
<tr>
<th>Vector</th>
<th>Description</th>
<th>Dimension</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>Height of contact of ball and tooth</td>
<td>0.6938</td>
<td>?</td>
</tr>
<tr>
<td>U2</td>
<td>Depth of contact of ball and tooth on side</td>
<td>0.375</td>
<td>?</td>
</tr>
<tr>
<td>U3</td>
<td>Depth of contact of ball and tooth on bottom</td>
<td>0.375</td>
<td>?</td>
</tr>
<tr>
<td>U4</td>
<td>Distance of contact of ball from tooth corner</td>
<td>0.69382</td>
<td>?</td>
</tr>
<tr>
<td>U5</td>
<td>Height of tooth corner from center of ratchet</td>
<td>3.39062</td>
<td>?</td>
</tr>
<tr>
<td>φ1</td>
<td>Angle of two vectors from center of ball to contact points with the tooth</td>
<td>99°</td>
<td>?</td>
</tr>
<tr>
<td>φ2</td>
<td>Angle of pawl with horizontal</td>
<td>0°</td>
<td>?</td>
</tr>
<tr>
<td>φ3</td>
<td>Angle between vectors entering and leaving ball on loop2</td>
<td>90°</td>
<td>?</td>
</tr>
</tbody>
</table>

Table 5.10 lists the eight unknown dependent dimensional tolerances that need to be solved for.

3-D Solutions With and Without Form Tolerances

The sensitivities for all dimensions and the values for the eight unknown tolerances were obtained using the same spreadsheet program used in the previous example problems.

The calculated tolerances for the dependent variables without form tolerances are given in table 5.12. The solutions were calculated for statistical and worst case results. These values show how well the assembly will fit together using the assigned tolerances on the independent (manufactured) dimensions and mating part types involved.
The same vector loop setup will be used for this analysis as was used in the analysis without form tolerances included. However, some form tolerances will be added and the two sets of compared.

Table 5.11 lists the form tolerances applied to this problem, where they were used and what their tolerance values are.

<table>
<thead>
<tr>
<th>Form Tolerance (Joint Type used)</th>
<th>Feature</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRO (Sph Slider)</td>
<td>Profile of sphere</td>
<td>0.002 (Tran)</td>
</tr>
<tr>
<td>PRO (Sph Slider)</td>
<td>Profile of bottom edge of tooth</td>
<td>0.002</td>
</tr>
<tr>
<td>PER (Rigid)</td>
<td>Perpendicularity of base and support</td>
<td>N/A</td>
</tr>
<tr>
<td>POS (Revolute)</td>
<td>Position of hole for pawl hinge</td>
<td>0.005</td>
</tr>
<tr>
<td>CON (Revolute)</td>
<td>Hinge of ratchet</td>
<td>0.002</td>
</tr>
</tbody>
</table>

The results from the spreadsheet calculations with and without form tolerances are given in table 5.12 below.
Table 5.12 Comparison of Resultant Tolerances

<table>
<thead>
<tr>
<th>dU</th>
<th>Stat w/o</th>
<th>Stat w/</th>
<th>WC w/o</th>
<th>WC w/</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>0.0124</td>
<td>0.0126</td>
<td>0.014</td>
<td>0.0164</td>
</tr>
<tr>
<td>U2</td>
<td>0.0154</td>
<td>0.0754</td>
<td>0.03</td>
<td>0.1087</td>
</tr>
<tr>
<td>U3</td>
<td>0.0154</td>
<td>0.0754</td>
<td>0.03</td>
<td>0.1087</td>
</tr>
<tr>
<td>U4</td>
<td>0.0124</td>
<td>0.0126</td>
<td>0.014</td>
<td>0.0164</td>
</tr>
<tr>
<td>U5</td>
<td>0.0172</td>
<td>0.0221</td>
<td>0.0428</td>
<td>0.0698</td>
</tr>
<tr>
<td>φ1</td>
<td>1.0027</td>
<td>1.0027</td>
<td>1.0027</td>
<td>1.0027</td>
</tr>
<tr>
<td>φ2</td>
<td>0.0</td>
<td>0.470</td>
<td>0.0</td>
<td>0.470</td>
</tr>
<tr>
<td>φ3</td>
<td>0.0754</td>
<td>0.0773</td>
<td>0.14177</td>
<td>0.1667</td>
</tr>
</tbody>
</table>

When the results are compared, it can again be seen that form tolerances are an integral part of the design and manufacturing process.

5.4 Linear Positioner Problem

This fourth and last example problem is taken from a hard-disk drive head positioning mechanism. In order to be of any use in a disk drive, the head must be able to be positioned anywhere on the disk with a high degree of accuracy. This is a good problem to show why tolerance analysis is so important and how form tolerances can affect the assembly. The head is moved along a set of rails to the desired position and so the wheel sets shown in Figure 5.10 become an important area to analyze for variations.
Figure 5.10  Linear Positioner

The number of loops required for this problem is determined to be 5. Figures 5.11 through 5.13 illustrate the location of these vector loops. Figure 5.13 also shows the applied form tolerances.
Figure 5.11 Vector Loops 1, 2 and 3 for Linear Positioner

Figure 5.12 Vector Loop 4 for Linear Positioner
Because of the difficulty in showing all the loop vectors in loop 5, only the ones unique to loop 5 are shown. The vectors not shown for loop 5 are the same as vectors a, u1, b, c, d, and e for loop 1 and vectors c, b, u3, and a (in that order) for loop 2.

The independent (or manufactured) dimensions and the dependent (assembly) dimensions are listed in tables 5.13 and 5.14 respectively.
<table>
<thead>
<tr>
<th>Vector</th>
<th>Description</th>
<th>Dimension</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>Radius of railR</td>
<td>0.80</td>
<td>0.0002</td>
</tr>
<tr>
<td>b1</td>
<td>Radius of wheel1, set1</td>
<td>0.80</td>
<td>0.002</td>
</tr>
<tr>
<td>c1</td>
<td>Length of wheel1, set1 axel</td>
<td>0.30</td>
<td>0.0012</td>
</tr>
<tr>
<td>d1</td>
<td>Distance of wheel1, set1 axel to front edge</td>
<td>1.20</td>
<td>0.003</td>
</tr>
<tr>
<td>e1</td>
<td>Distance of wheel1, set1 axel to right side edge</td>
<td>1.0</td>
<td>0.003</td>
</tr>
<tr>
<td>f1</td>
<td>Distance of wheel2, set1 axel to right side edge</td>
<td>1.0</td>
<td>0.003</td>
</tr>
<tr>
<td>g1</td>
<td>Distance of wheel2, set1 axel to front edge</td>
<td>1.20</td>
<td>0.003</td>
</tr>
<tr>
<td>h1</td>
<td>Length of wheel2, set1 axel</td>
<td>0.30</td>
<td>0.0012</td>
</tr>
<tr>
<td>k1</td>
<td>Radius of wheel2, set1</td>
<td>0.80</td>
<td>0.002</td>
</tr>
<tr>
<td>m1</td>
<td>Radius of railR</td>
<td>0.80</td>
<td>0.0002</td>
</tr>
<tr>
<td>a2</td>
<td>Radius of railR</td>
<td>0.80</td>
<td>0.0002</td>
</tr>
<tr>
<td>b2</td>
<td>Radius of wheel1, set2</td>
<td>0.80</td>
<td>0.002</td>
</tr>
<tr>
<td>c2</td>
<td>Length of wheel1, set2 axel</td>
<td>0.30</td>
<td>0.0012</td>
</tr>
<tr>
<td>d2</td>
<td>Distance of wheel1, set2 axel to front edge</td>
<td>1.20</td>
<td>0.003</td>
</tr>
<tr>
<td>e2</td>
<td>Distance of wheel1, set2 axel to right side edge</td>
<td>1.0</td>
<td>0.003</td>
</tr>
<tr>
<td>f2</td>
<td>Distance of wheel2, set2 axel to right side edge</td>
<td>1.0</td>
<td>0.003</td>
</tr>
<tr>
<td>g2</td>
<td>Distance of wheel2, set2 axel to front edge</td>
<td>1.20</td>
<td>0.003</td>
</tr>
<tr>
<td>h2</td>
<td>Length of wheel2, set2 axel</td>
<td>0.30</td>
<td>0.0012</td>
</tr>
<tr>
<td>k2</td>
<td>Radius of wheel2, set2</td>
<td>0.80</td>
<td>0.002</td>
</tr>
<tr>
<td>m2</td>
<td>Radius of railR</td>
<td>0.80</td>
<td>0.0002</td>
</tr>
<tr>
<td>a3</td>
<td>Radius of railL</td>
<td>0.80</td>
<td>0.0002</td>
</tr>
<tr>
<td>b3</td>
<td>Radius of wheel2, set3</td>
<td>0.80</td>
<td>0.002</td>
</tr>
<tr>
<td>c3</td>
<td>Length of wheel2, set3 axel</td>
<td>0.30</td>
<td>0.0012</td>
</tr>
<tr>
<td>d3</td>
<td>Distance of wheel2, set3 axel to front edge</td>
<td>1.20</td>
<td>0.003</td>
</tr>
<tr>
<td>e3</td>
<td>Distance of wheel2, set3 axel to right side edge</td>
<td>1.0</td>
<td>0.003</td>
</tr>
<tr>
<td>f3</td>
<td>Distance of wheel1, set3 axel to right side edge</td>
<td>1.0</td>
<td>0.003</td>
</tr>
<tr>
<td>g3</td>
<td>Distance of wheel1, set3 axel to front edge</td>
<td>1.20</td>
<td>0.003</td>
</tr>
<tr>
<td>h3</td>
<td>Length of wheel1, set3 axel</td>
<td>0.30</td>
<td>0.0012</td>
</tr>
<tr>
<td>k3</td>
<td>Radius of wheel1, set3</td>
<td>0.80</td>
<td>0.002</td>
</tr>
<tr>
<td>m3</td>
<td>Radius of railL</td>
<td>0.80</td>
<td>0.0002</td>
</tr>
<tr>
<td>g4</td>
<td>Distance from right edge to bottom datum</td>
<td>4.826</td>
<td>0.003</td>
</tr>
<tr>
<td>h4</td>
<td>Distance from bottom datum to left edge</td>
<td>4.826</td>
<td>0.003</td>
</tr>
<tr>
<td>r4</td>
<td>Distance from railL center to railR center</td>
<td>8.48529</td>
<td>0.010</td>
</tr>
<tr>
<td>f5</td>
<td>Distance from front edge to wheel1, set2 axel</td>
<td>7.80</td>
<td>0.010</td>
</tr>
<tr>
<td>g5</td>
<td>Distance from right edge to wheel1, set2 axel</td>
<td>1.0</td>
<td>0.003</td>
</tr>
</tbody>
</table>
Table 5.14  Dependent Dimensions for Linear Positioner

<table>
<thead>
<tr>
<th>Vector</th>
<th>Description</th>
<th>Dimension</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>Half the thickness of wheel1,set1</td>
<td>0.30</td>
<td>?</td>
</tr>
<tr>
<td>U2</td>
<td>Half the thickness of wheel2,set1</td>
<td>0.30</td>
<td>?</td>
</tr>
<tr>
<td>U3</td>
<td>Half the thickness of wheel1,set2</td>
<td>0.30</td>
<td>?</td>
</tr>
<tr>
<td>U4</td>
<td>Half the thickness of wheel2,set2</td>
<td>0.30</td>
<td>?</td>
</tr>
<tr>
<td>U5</td>
<td>Half the thickness of wheel2,set3</td>
<td>0.30</td>
<td>?</td>
</tr>
<tr>
<td>U6</td>
<td>Half the thickness of wheel1,set3</td>
<td>0.30</td>
<td>?</td>
</tr>
<tr>
<td>U7</td>
<td>Distance along railR from wheel set3 to wheel set1</td>
<td>4.30</td>
<td>?</td>
</tr>
<tr>
<td>U8</td>
<td>Distance along railR from wheel set2 to wheel set1</td>
<td>6.60</td>
<td>?</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>Angle between horizontal and vector from center of railR to contact of wheel1,set1</td>
<td>45°</td>
<td>?</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>Angle between horizontal and vector from center of railR to contact of wheel2,set1</td>
<td>45°</td>
<td>?</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>Angle between horizontal and vector from center of railR to contact of wheel1,set2</td>
<td>45°</td>
<td>?</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>Angle between horizontal and vector from center of railR to contact of wheel2,set2</td>
<td>45°</td>
<td>?</td>
</tr>
<tr>
<td>$\phi_5$</td>
<td>Angle between horizontal and vector from center of railR to contact of wheel2,set3</td>
<td>45°</td>
<td>?</td>
</tr>
<tr>
<td>$\phi_6$</td>
<td>Angle between horizontal and vector from center of railR to contact of wheel1,set3</td>
<td>45°</td>
<td>?</td>
</tr>
</tbody>
</table>
Table 5.14 lists the 14 unknown dependent dimensional tolerances that need to be solved for.

**3-D Solutions With and Without Form Tolerances**

The sensitivities for all dimensions and the values for the 14 unknown tolerances were again obtained using the spreadsheet program used in the previous example problems.

The calculated tolerances for the dependent variables are given in table 5.16. The solutions were calculated for statistical and worst case results.

The same vector loop setup will be used for this analysis as was used in the previous analysis. Again, form tolerances will be included and the values computed. Table 5.15 lists the form tolerances applied to this problem, where they were used and what their tolerance values are.

**Table 5.15  Form Tolerances for Linear Positioner Problem**

<table>
<thead>
<tr>
<th>Form Tolerance (Joint Type used)</th>
<th>Feature</th>
<th>Tolerance(±)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RUN (Crossed Cyl)</td>
<td>Wheels for all loops (10 total)</td>
<td>0.001</td>
</tr>
<tr>
<td>STR (Crossed Cyl)</td>
<td>Rails for all loops (6 total)</td>
<td>0.001</td>
</tr>
</tbody>
</table>

When the two results are compared, it can be seen (table 5.16) that form tolerances are an integral part of the design and manufacturing process.
Table 5.16  Comparison of Resultant Tolerances

<table>
<thead>
<tr>
<th>$dU$</th>
<th>Stat w/o</th>
<th>Stat w/</th>
<th>WC w/o</th>
<th>WC w/</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>0.00385</td>
<td>0.00392</td>
<td>0.01201</td>
<td>0.01373</td>
</tr>
<tr>
<td>U2</td>
<td>0.00381</td>
<td>0.00406</td>
<td>0.0064</td>
<td>0.0084</td>
</tr>
<tr>
<td>U3</td>
<td>0.00314</td>
<td>0.00322</td>
<td>0.0092</td>
<td>0.01057</td>
</tr>
<tr>
<td>U4</td>
<td>0.00381</td>
<td>0.00406</td>
<td>0.0064</td>
<td>0.0084</td>
</tr>
<tr>
<td>U5</td>
<td>0.00381</td>
<td>0.00406</td>
<td>0.0064</td>
<td>0.0084</td>
</tr>
<tr>
<td>U6</td>
<td>0.00521</td>
<td>0.0053</td>
<td>0.01481</td>
<td>0.0169</td>
</tr>
<tr>
<td>U7</td>
<td>0.00424</td>
<td>0.00424</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>U8</td>
<td>0.01044</td>
<td>0.01044</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>φ1</td>
<td>0.03808</td>
<td>0.0397</td>
<td>0.13267</td>
<td>0.16132</td>
</tr>
<tr>
<td>φ2</td>
<td>0.03808</td>
<td>0.0397</td>
<td>0.13267</td>
<td>0.16132</td>
</tr>
<tr>
<td>φ3</td>
<td>0.03808</td>
<td>0.0397</td>
<td>0.13267</td>
<td>0.16132</td>
</tr>
<tr>
<td>φ4</td>
<td>0.03808</td>
<td>0.0397</td>
<td>0.13267</td>
<td>0.16132</td>
</tr>
<tr>
<td>φ5</td>
<td>0.03808</td>
<td>0.0397</td>
<td>0.13267</td>
<td>0.16132</td>
</tr>
<tr>
<td>φ6</td>
<td>0.03808</td>
<td>0.0397</td>
<td>0.13267</td>
<td>0.16132</td>
</tr>
</tbody>
</table>

The effect of form tolerances in an assembly has been shown in the four example problems presented in this chapter. Although the effect will vary for each assembly, depending on the geometry and applicable form tolerances, form tolerances do play a considerable role in the design and manufacturing world.
Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Research Contributions

This thesis advances the abilities of the 3-dimensional tolerance modeler and analysis software to include the American National Standards Institute (ANSI) geometric form tolerances. This was accomplished through evaluating and refining the work done by Goodrich [1991] by carefully cataloging all 132 joint-form tolerance combinations he layed out. These 132 cases were subsequently reduced to eight basic cases. This was the key step in developing a systematic approach to obtaining sensitivities. The author's contributions are listed below followed by a brief discussion of each.

The objective of the research for this thesis was to extend present systems for tolerance analysis of mechanical assemblies including:

1. development of engineering models for geometric form and orientation variations

2. describing their effects on 3-D multiple part assemblies

3. demonstrating a systematic procedure for performing statistical and worst case analysis including form and orientation variations

The 3-D models for geometric form and orientation variations were closely scrutinized and extensively refined to provide accurate information on form variations. These revised models are now ready to be implemented in the 3-D tolerance modeling systems. The revisions included refining the joint/form tolerance matrix presented by Goodrich [1991]. This matrix was looked at joint-by-joint resulting in a more structured format.

Multiple part assemblies can be more accurately modeled and analyzed using the engineering models for geometric form and orientation to describe how they accumulate statistically and propagate kinematically in mechanical assemblies. This ability to model and analyze multiple part assemblies was the major force behind this research.
Being able to include statistical analysis and worst case analysis was accomplished by the ability to create accurate 3-D form matrices that can be included in the analysis equations developed previously. The key to this was a systematic procedure for obtaining tolerance sensitivities and including them in the linearized engineering model suitable for automating much of the process to assist the design engineer in formulating a correct model.

Another contribution provided in this thesis was an updated neutral file. This file is the link between the tolerance modeler and the 3-D analysis program. The neutral file standard was revised to include appropriate information required for correct analysis including form tolerances.

The Fortran code for the extracting information from the CATIA model file and generating a CATS Neutral File was written and implemented in the tolerance modeler for CATIA. The neutral file routines are currently being used in both the mainframe and RISC based versions of CATIA.

6.2 Recommendations for Future Work

Much of the work completed in this thesis is providing a solid basis for 3-D tolerance modelling and analysis. There are many areas open for further research, some of which are presented below.

1. Now that the engineering models for each type of form variation have been completely defined for all twelve kinematic joints, and the procedure for obtaining the sensitivities outlined, these features can be implemented into the CATS 3-D analysis code.

2. Automatic loop generation algorithms for 3-D for both closed and open loops.

3. Develop a direct link between the 3-D modeler and the tolerance analysis systems. This would enable the designer to accomplish analysis and design iterations with a minimum of effort. Currently the designer must enter the tolerance model information in the CAD model and then extract that information (via a neutral file) to feed to the analysis package. Thus, linking the two would greatly simplify the task.

4. Incorporate a method for graphically viewing the analysis results directly in the CAD model to give graphical feedback to the designer.
5. Develop a surface/feature recognition algorithm that would enable the tolerance modeler to automatically determine mating surface types and subsequently select the appropriate joint types for the modeling.

6. Develop 3-D tolerance modelers for other CAD systems.

7. Incorporate form tolerance analysis into the 3-D analysis package.

8. Provide for kinematic size tolerance assignments in the tolerance modeler.

9. Investigate the possibility for geometric and tolerance associativity so that changes in the CAD model would also modify the CATS assembly tolerance model.

10. Allow for equivalencing capabilities to automatically recognize and link equivalent vectors and angles in different loops.
REFERENCES


References


Shapiro, S., and A. Gross (1981), Statistical Modeling Techniques, Marcel Dekker.


APPENDIX

A. DATA STRUCTURE FOR CATS.BYU ASSEMBLY TOLERANCE MODELER FOR CATIA

B. OBJECT DESCRIPTORS

C. NEUTRAL FILE DEFINITION

D. CATIA API FUNCTION CALLS USED IN CATS ROUTINES

E. NEUTRAL FILE FORTRAN ROUTINES
A. DATA STRUCTURE FOR CATS.BYU ASSEMBLY TOLERANCE MODELER FOR CATIA

Guide for reading data structures:

- Means first sub-level association to object.
- Means second sub-level association to object.

<table>
<thead>
<tr>
<th>LOOP:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Vectors #1, #2, etc. (loop application element #1)</td>
</tr>
<tr>
<td>• either Joint, DRF or FDTM (vector application element #1)</td>
</tr>
<tr>
<td>• either Joint, DRF or FDTM (vector application element #2)</td>
</tr>
<tr>
<td>• vector symbol (vector application element #3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VECTOR:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Joint</td>
</tr>
<tr>
<td>• datum Path #1 (joint application element #1)</td>
</tr>
<tr>
<td>• datum Path #2 (joint application element #2)</td>
</tr>
<tr>
<td>• joint symbol (joint application element #3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VECTOR:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• DRF</td>
</tr>
<tr>
<td>• CATIA solid (DRF application element #1)</td>
</tr>
<tr>
<td>• DRF symbol (DRF application element #2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VECTOR:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• FDTM</td>
</tr>
<tr>
<td>• CATIA solid (FDTM application element #1)</td>
</tr>
<tr>
<td>• FDTM symbol (FDTM application element #2)</td>
</tr>
</tbody>
</table>
### JOINT:

<table>
<thead>
<tr>
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<th>Description</th>
<th>Comment</th>
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</thead>
<tbody>
<tr>
<td>#1</td>
<td>Datum Path</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• DRF</td>
<td>(datum path application element)</td>
</tr>
<tr>
<td>#2</td>
<td>• FDTM #1 or joint</td>
<td>(datum path application element)</td>
</tr>
<tr>
<td>#3</td>
<td>• FDTM #2 or joint</td>
<td>(datum path application element)</td>
</tr>
<tr>
<td></td>
<td>• FDTM #3, #4, etc.</td>
<td>(datum path app. element #4, etc.)</td>
</tr>
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</table>

### FEATURE CONTROL:

<table>
<thead>
<tr>
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<th>Comment</th>
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</thead>
<tbody>
<tr>
<td>Solid</td>
<td>(feature control app. element #1)</td>
</tr>
<tr>
<td>DRF</td>
<td>(feature control app. element #2)</td>
</tr>
<tr>
<td>FDTM #1 or joint</td>
<td>(feature control app. element #3)</td>
</tr>
</tbody>
</table>
### B. OBJECT DESCRIPTORS

<table>
<thead>
<tr>
<th>Object Type</th>
<th>Descriptor</th>
<th>Variable Type</th>
<th>API Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loop</td>
<td>Type</td>
<td>I*4</td>
<td>I4(1)</td>
<td>ILOP</td>
</tr>
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<td>Vector</td>
<td>None</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Joint</td>
<td>Type</td>
<td>I*4</td>
<td>I4(1)</td>
<td>ILOC</td>
</tr>
<tr>
<td>Loc</td>
<td>R*8</td>
<td>R8(1-3)</td>
<td>ILOC</td>
<td></td>
</tr>
<tr>
<td>Axis #1</td>
<td>R*8</td>
<td>R8(1-3)</td>
<td>IIAXS</td>
<td></td>
</tr>
<tr>
<td>Axis #2</td>
<td>R*8</td>
<td>R8(4-6)</td>
<td>IIAXS</td>
<td></td>
</tr>
<tr>
<td>DRF</td>
<td>Type</td>
<td>I*4</td>
<td>I4(1)</td>
<td>ILOC</td>
</tr>
<tr>
<td>Loc</td>
<td>R*8</td>
<td>R8(1-3)</td>
<td>ILOC</td>
<td></td>
</tr>
<tr>
<td>Axis</td>
<td>R*8</td>
<td>R8(1-3)</td>
<td>IDAXS</td>
<td></td>
</tr>
<tr>
<td>FDTM</td>
<td>Type</td>
<td>I*4</td>
<td>I4(1)</td>
<td>ILOC</td>
</tr>
<tr>
<td>Loc</td>
<td>R*8</td>
<td>R8(1-3)</td>
<td>ILOC</td>
<td></td>
</tr>
<tr>
<td>Axis</td>
<td>R*8</td>
<td>R8(1-3)</td>
<td>IDAXS</td>
<td></td>
</tr>
<tr>
<td>Datum Path</td>
<td>None</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>I4(1)</td>
<td>ILOC</td>
</tr>
<tr>
<td>Loc</td>
<td>R*8</td>
<td>R8(1-3)</td>
<td>ILOC</td>
<td></td>
</tr>
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<td>C*8</td>
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<td></td>
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</tbody>
</table>
C. NEUTRAL FILE DEFINITION

FILE>> WORKFILE
DESCRIPTION OF DATA FILE:
$Any description entered will be preceded by "$" on the next lines. The data $file description will precede the first assembly block.
--------------- ASSEMBLY: GEOMSTACK ( )-------------------------------------
DESCRIPTION: Sample neutral data file for transmitting tolerance stack data $ from a CAD system to CATS.BYU.
$ 1. File specification: ASCII, 80 character, formatted sequentially, 80th $ character never used.
$ 2. The first line is a file header containing file name and creation info.
$ 3. Each block begins with a header line (----) containing the block type $ (e.g. LOOP:) and 14 character code name required by CATS.
$ 4. Every block can have its own description. Any number of continuation $ lines may be added, each preceded by "$ ".
$ 5. Each record type is preceded by a unique 3 character mnemonic label.
$ 6. Each data item has a bracket behind it ( ) for a "change flag". Any data item which has been changed by CATS is flagged by a (/).
$ Data which may not be changed (vendor-supplied or design $ requirement) may be flagged by a (*).
$ 7. Label Record Type:
  ---- Data Block Separator.
  ANM Assembly Name
  ANU Assembly No.
  DRA Drawing No.
  MOD List the related assembly loops defining a single model.
  PNM Part Name
  PNU Part Number
  PAR Part Dimension
  COS Part Cost
  DES Description
  TEX Non-Data Record (ignored)
  $ Continuation
$ 8. Format Specifications:
  AAAAA ASCII field
  XXX.XX Real number field
  (A) Change field
  TEX Column headings and separators

ANN ASSM NAME: STACKED GEOMETRIC SHAPES
ANU ASSM NO: ASNO-342
DRAWING NO.: 0027-943
MODEL: GLOOP1 ( ) GLOOP2 ( ) GLOOP3 ( )
$Continuation lines may be necessary depending on the model type, etc.
--------------- SPECIFICATIONS: ---------------------------------------------
TEX TYPE REF-JOI/DIM 2ND-JOI/DIM BAS DIM MAX TOL MIN TOL
SPE CLR JOINT5 JOINT4 19.058 0.322 -0.322
$ SPE Assembly Specs (clearance)
$ SPECIFICATION TYPES:
$ CLR 3-D clearance
$ ANG Angle between 2 dimensions
$ ORI Orientation from global axis to dimension
$ POS Position from global reference
$ Specification block format and description.
$TEX TYPE REF-JOI/DIM 2ND-JOI/DIM BAS DIM MAX TOL MIN TOL
$TEX AAA AAAAAAAAA (A) AAAAAAAAA (A) XXX.XXX(A) XXX.XXX(A) XXX.XXX (A)
$ ^ ^ Basic dimension of mag/angle.
$ ^ ^ 2nd joint for clearance specification.
$ ^ ^ 2nd dimension for angle/orientation specification.
$ Reference joint for clearance specification.
$ Reference dimension for angle specification.
$ Reference global axis for orientation specification.

DESCRIPTION: Descriptions can be entered for each part in the assembly. All parts are listed after the assembly description and before the first loop with any description which may have been entered.

PART: P2

PNM: Cylinder
PNM: PTN-342
DRAWING NO.: 0027-943
PART DIMENSIONS:

PART: P3

DESCRIPTION:

PART NAME:
PART NO:
DRAWING NO:
PART DIMENSIONS:

PART: P1

DESCRIPTION:

PART NAME:
PART NO:
DRAWING NO:
PART DIMENSIONS:

$ DATUM LIST:

$ Datum block format and description:
$TEX DATUM NAME TYPE X Y Z
$AA AAAAAAAAAA(A) AAAAA(A) XXX.XXXXX XXX.XXXXX XXX.XXXXX(A)
$TEX

$ DRF Datum Reference Frame
$ AXI Local axis info for joint data.
$ DAT Feature Datum
$ AXI Axis info for independent angle rotation.
$ DATUM TYPES:
$ Rectangular:
$ RECDRF
$ RECDAT
$ CYLINDRICAL Center
$ CYLDRF
$ CYLDAT
$ SPHERICAL Center
$ SPHDFR
$ SPHDAT

TEX

DESCRIPTION:

TEX DATUM NAME TYPE X Y Z
DRF DATUM3 ( ) RECTAN ( ) 5.0000000 5.0000000 0.0000000 ( )
DRF DATUM2 ( ) RECTAN ( ) 6.7909300 8.4821892 0.0000000 ( )
DRF DATUM1 ( ) CYLCEN ( ) 11.520000 23.718240 0.0000000 ( )
AXI 0.0000000 0.0000000 1.0000000 ( )
DAT DATUM10 ( ) RECTAN ( ) 33.125000 5.0000000 0.0000000 ( )
DAT DATUM8 ( ) RECTAN ( ) 8.9050000 5.0000000 0.0000000 ( )
DAT DATUM5 ( ) RECTAN ( ) 5.0000000 15.047294 0.0000000 ( )

$ JOINT LIST:

$ Joint block format and description:
$TEX JOINT NAME TYPE X Y Z
$AA AAAAAAAAAA(A) AAAAA(A) XXX.XXXXX(A) XXX.XXXXX(A) XXX.XXXXX(A)
$ Global location of the joint.
$TEX

$ JOI Joint Data
$ JOINT TYPES:
Appendix

<table>
<thead>
<tr>
<th>RIGID</th>
<th>Rigid</th>
<th>REVOLU</th>
<th>Revolution (pin joint)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CYLIND</td>
<td>Cylindrical</td>
<td>UJOINT</td>
<td>U-Joint</td>
</tr>
<tr>
<td>PRISM</td>
<td>Prism</td>
<td>BALLJT</td>
<td>Ball Joint</td>
</tr>
<tr>
<td>PLANAR</td>
<td>Planar</td>
<td>EDGSLI</td>
<td>Edge Slider</td>
</tr>
<tr>
<td>CYLSLI</td>
<td>Cylind. Slider</td>
<td>PNTSLL</td>
<td>Point Slider</td>
</tr>
<tr>
<td>SPHSLI</td>
<td>Spherical Slider</td>
<td>CRSCYL</td>
<td>Cross Cylinder</td>
</tr>
<tr>
<td>AXI</td>
<td>Local axis info for joint data.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AXIS TYPES:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PLANAR</td>
<td>Planar</td>
<td>AXIAL</td>
<td>Axial</td>
</tr>
<tr>
<td>NORMAL</td>
<td>Normal</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**DESCRIPTION:** Each loop is stored as a table in a block of 80 character records.

**LABEL** RECORD TYPE | LABEL RECORD TYPE
---|---|---|---|---

**FORMAT SPECIFICATIONS FOR VECTOR AND JOINT LISTS:**

**TEXT**

---

**TABLE:** 1 TYPE: HOB X: -1.347656 Y: 0.131836 TEXTSIZE: 0.164

---

**TEXT**

---

**NODE**: PART/1-2 BAS DIM MAX TOL MIN TOL DEF

---

**NODE**: PART/1-2 BAS DIM MAX TOL MIN TOL DEF

---

**NODE**: PART/1-2 BAS DIM MAX TOL MIN TOL DEF
Appendix  96

NOD  JOINT4
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  180.00000( )  0.00000( )  0.00000( )  ( )
DIM  P1/0-4  ( )  18.718200( )  0.000500( )  0.000500( )  DEP( )
NOD  JOINT0

====================================================================
LOOP:  GLOOP2
( )
DESCRIPTION:  Loop specs may be defined in terms of unequal tolerances as shown.
$ Keywords in the record tell CATS which format is being used.

TABLE:  2  TYPE:  HGR  X:  0.550781  Y:  5.077149  TEXTSIZE:  0.164
NOD  JOINT3
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  -164.7239( )  0.00000( )  0.00000( )  DEP( )
DIM  P3/3-6  ( )  6.805000( )  0.075000( ) -0.075000( )  ( )
NOD  JOINT6
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  90.00000( )  0.00000( )  0.00000( )  ( )
DIM  P3/6-7  ( )  2.189400( )  0.00000( )  0.00000( )  ( )
NOD  JOINT7
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  -105.2761( )  0.00000( )  0.00000( )  ( )
DIM  P1/7-8  ( )  4.060000( )  0.150000( ) -0.150000( )  ( )
NOD  JOINT8
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  -90.00000( )  0.00000( )  0.00000( )  DEP( )
DIM  P1/8-9  ( )  3.665000( )  0.125000( )  0.00000( )  DEP( )
NOD  JOINT9
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  270.0000( )  0.00000( )  0.00000( )  ( )
DIM  P1/3-9  ( )  10.04770( )  0.00000( )  0.00000( )  DEP( )
NOD  JOINT3

====================================================================
LOOP:  GLOOP3
( )
DESCRIPTION:  The basic dimensions are determined by symmetric or nonsymmetric
$ tolerancing.  Keywords in the record tell CATS which format is being used.

TABLE:  2  TYPE:  HGR  X:  0.550781  Y:  5.077149  TEXTSIZE:  0.164
NOD  JOINT15
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  -105.2761( )  0.00000( )  0.00000( )  DEP( )
DIM  P1/11-15  ( )  10.67500( )  0.125000( ) -0.125000( )  ( )
NOD  JOINT11
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  180.00000( )  0.00000( )  0.00000( )  DEP( )
DIM  P1/11-12  ( )  4.060000( )  0.150000( ) -0.150000( )  ( )
NOD  JOINT12
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  90.00000( )  0.00000( )  0.00000( )  DEP( )
DIM  P1/7-12  ( )  22.44000( )  0.350000( ) -0.350000( )  ( )
NOD  JOINT7
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
ZRD  15.27610( )  0.00000( )  0.00000( )  DEP( )
DIM  P3/7-14  ( )  2.189400( )  0.00000( )  0.00000( )  ( )
NOD  JOINT14
YRD  0.00000( )  0.00000( )  0.00000( )  ( )
## Appendix 97

ZRO: -180.000( ) 0.000000( ) 0.000000( ) DEX( )
DIM P3/14-15 ( ) 27.29650( ) 0.000000( ) 0.000000( ) ( )

**NOD JOINT15**

**== FEATURE TOL LIST: ==**

**DESCRIPTION:**

<table>
<thead>
<tr>
<th>S</th>
<th>FEATURE CONTROLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>FLA Flatness</td>
</tr>
<tr>
<td>S</td>
<td>STR Straightness</td>
</tr>
<tr>
<td>S</td>
<td>PAR Parallelism</td>
</tr>
<tr>
<td>S</td>
<td>PER Perpendicularity</td>
</tr>
<tr>
<td>S</td>
<td>ANG Angularity</td>
</tr>
<tr>
<td>S</td>
<td>TRU True Position</td>
</tr>
</tbody>
</table>

**SFEA AAAAAAAAA AAA(A) XXX.XXXX(A) XXX.XXXX(A) XXX.XXXX(A) XXX.XXXX(A)**

<table>
<thead>
<tr>
<th>TEX JOINT NAME</th>
<th>PART</th>
<th>TYPE</th>
<th>TOLERANCE</th>
<th>CHAR.LENGTH</th>
<th>CHAR.LENGTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA JOINT1</td>
<td>PART1</td>
<td>ROU</td>
<td>0.0500000( ) 0.000000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT2</td>
<td>PART2</td>
<td>PAR</td>
<td>0.0000000( ) 0.0060000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT2</td>
<td>PART2</td>
<td>ANG</td>
<td>0.0000000( ) 0.0060000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT3</td>
<td>PART3</td>
<td>ANG</td>
<td>0.0040000( ) 0.0000000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT3</td>
<td>PART3</td>
<td>PER</td>
<td>0.0000000( ) 0.0500000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT4</td>
<td>PART4</td>
<td>PER</td>
<td>0.0000000( ) 0.0700000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT5</td>
<td>PART5</td>
<td>ROU</td>
<td>0.0000000( ) 0.0500000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT7</td>
<td>PART7</td>
<td>PAR</td>
<td>0.0000000( ) 0.0200000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT7</td>
<td>PART7</td>
<td>PER</td>
<td>0.0000000( ) 0.0200000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT9</td>
<td>PART9</td>
<td>PER</td>
<td>0.0000000( ) 0.0100000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT9</td>
<td>PART9</td>
<td>PER</td>
<td>0.0000000( ) 0.0100000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT12</td>
<td>PART12</td>
<td>PER</td>
<td>0.0000000( ) 0.0400000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEA JOINT12</td>
<td>PART12</td>
<td>PAR</td>
<td>0.0000000( ) 0.0600000( ) 0.000000( )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**END**
D. CATIA API FUNCTION CALLS USED IN CATS ROUTINES

GCWCI3............Create a SPACE circle or arc passing through 3 points.
GCWLN..............Create a SPACE line by using two points.
GCWSCO.............Create a cone of a frustrum of a cone.
GCWSCY.............Create a cylinder (possibly hollow).
GCWSPH.............Create a sphere (possibly hollow).
GCWSPL.............Create a parallelepiped.
GCWTVE.............Create a temporary vector in SPACE mode (/w arrow).
GGCLST.............Close the currently open structure.
GGOPST.............Open a structure.
GIADDEL............Delete application data previously saved in storage area.
GIAMOD.............Modify application data previously saved in storage area.
GIAREA.............Read appl. data previously saved in a storage area.
GIAWRI.............Write appl. data in a storage area.
GICCOL.............Modify the color of an element.
GICIDE.............Change the identifier of an element.
GICTEM.............Called just prior to creating a temp. geometric element.
GICTMM.............Activate/deactivate option for creating temp. geom. element.
GIDENT.............Delete a temporary geometric element.
GIDTTEXT...........Delete a SPACE text.
GIERAS$............Erase one of a list of model entities.
GIHHTLT............Modify the HIGHLIGHT temporary attribute of an element.
GIHIDE.............Read the identifier of an element.
GIHMAT.............Read description block associated to an element.
GIRTOL.............Read the five tolerances of a model.
GIRTPS.............Read the type of an entity.
GIRUNI.............Read the unit of a model.
GIWTEX.............Associate text to a SPACE element.
GIWTP.............Create a SPACE point.
GMACEL.............Create an application element (1ary type = 103).
GMALNK.............Add a link to an application element.
GMAREL.............Read the number of links on an application element.
GMARLN.............Read the elements linked to an application element.
GMASEL.............Search for application elements of a given type in an application set.
GMASLN.............Scan the application elements to which an element is linked.
GMASST.............Search for an application set of a given type.
GMAUNL.............Delete a link on an application element.
GMDRDV.............Read a description associated to an element.
GMDWRI.............Associate a description to an element.
GMGCSE.............Create a SPACE geometric element (1ary type = 29).
GMICGI.............Check that a panel selection was made at requested level.
GMKEY.............Retrieve data resulting from a keyboard entry.
GMKILG.............Retrieve data resulting from a keyboard entry.
GMISEL.............Retrieve data resulting from a selection.
GMIYES.............Check that a YES interaction was performed at requested level.
GSCCLO.............Creates a SPACE circle. Defined by line & point not on line.
GSTCOA.............Creates a SPACE transformation (about a point).
GUEPAR.............Initialize or modify the error parameters used for writing full messages.
GUMVEC.............Returns a unit vector in the direction of the cross product.
GUSEND.............Delete one or all stacks. The stack is deallocated.
GUSINF.............Retreive data relating to a stack.
GUSINI............. Create a stack.
GUSPOP............. Unstack a component. Component on top of stack is removed.
GUSPUS............. Stack a component. Component is added on top of stack.
GUSREA............. Read a stack component. Comp. stacked 1st has number one.
E. NEUTRAL FILE FORTRAN Routines

NEUTCT:
Main neutral file routine. Opens neutral file for writing, creates data stacks for each application element and calls each subroutine in order for writing to Neutral File.

NEUHDR:
Writes header lines to Neutral File.

NEUSPC:
Searches for SPECIFICATION data in model file and writes appropriate information to the Neutral File.

NEUPRT:
Searches for PART data in model file and writes appropriate information to the Neutral File.

NEUDTM:
Searches for DATUM data (both DTM and FDTM) in model file and writes appropriate information to the Neutral File.

NEUJNT:
Searches for JOINT data and writes appropriate information to the Neutral File.

NEUFCT:
Searches for FEATURE CONTROL data in the model file and writes appropriate information to the Neutral File.

NEULOP:
Creates data stacks for storing vector addresses in and writes LOOP header into the Neutral File. Calls Neulop1.

NEULOP1:
Searches for VECTOR data and writes appropriate information to the Neutral File.
Integrating Geometric Form Variations into Tolerance Analysis of 3-D Assemblies

Kerry Ryan Ward
Department of Mechanical Engineering
M.S. Degree, September 1992

ABSTRACT

Tolerance analysis is a vital part of any design project since tolerances directly affect cost, quality and performance. Being able to determine the probability of successfully assembling parts and meeting engineering requirements before any parts are manufactured is crucial to the success of a product in today's highly competitive market place. Having the ability to perform tolerance analyses on the computer using a CAD model data base can assure an efficient and accurate tolerancing effort.

There are three main sources of variation in mechanical assemblies: 1) Dimensional, 2) Form or feature, and 3) kinematic variations. Form variations arise from variations in shape, orientation or location as described by the geometric dimensioning and tolerancing standard, ANSI Y14.5M. Form variations can have a significant effect on an assembly, since they can accumulate statistically and propagate kinematically the same dimensional variations.

The emphasis of this project is to expand the current computer-aided modeling system to accurately utilize form tolerances as defined by the ANSI Y14.5 standard and exchange this information with a 3-D tolerance analysis package. With all three variation sources accounted for, 3-D tolerance analysis can now be performed entirely using the computer.

COMMITTEE APPROVAL:

Kenneth W. Chase, Committee Chairman
Spencer P. Magleby, Committee Member
John N. Cannon, Graduate Coordinator
Integrating Geometric Form Variations into
Tolerance Analysis of 3-D Assemblies

A Thesis
Presented to the
Department of Mechanical Engineering
Brigham Young University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
Kerry Ward
August 1992
This thesis, by Kerry Ward, is accepted in its present form by the Department of Mechanical Engineering of Brigham Young University as satisfying the thesis requirement for the degree of Master of Science.

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