Generalized 3-D Tolerance Analysis of Mechanical Assemblies with Small Kinematic Adjustments

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Abstract

The Direct Linearization Method (DLM) for tolerance analysis of 3-D mechanical assemblies is presented. Vector assembly models are used, based on 3-D vector loops which represent the dimensional chains that produce tolerance stackup in an assembly. Tolerance analysis procedures are formulated for both open and closed loop assembly models. The method generalizes assembly variation models to include small kinematic adjustments between mating parts.

Open vector loops describe critical assembly features. Closed vector loops describe kinematic constraints for an assembly. They result in a set of algebraic equations which are implicit functions of the resultant assembly dimensions. A general linearization procedure is outlined, by which the variation of assembly parameters may be estimated explicitly by matrix algebra.

Solutions to an over-determined system or a system having more equations than unknowns are included. A detailed example is presented to demonstrate the procedures of applying the DLM to a 3-D mechanical assembly.

1. Introduction

The importance of tolerance analysis and the control of manufacturing variation have received increased recognition as manufacturing industries strive to improve productivity and the quality of their products. There is a realization that it is no longer acceptable to arbitrarily select the tolerances on engineering drawings, as the effects of tolerance assignment are far-reaching. Not only do the tolerances and variations affect the ability to assemble the final product, but also the production cost, process selection, tooling, set-up cost, required operator skills, inspection and gaging, and scrap and rework. The variations constrained or bounded by the tolerances also directly affect product performance and robustness of the design. And poorly performing products will eventually lose out in the marketplace.

Both engineering design and manufacturing personnel are concerned about the effects of tolerances. Engineers often assign tight tolerances to assure fit and function of their designs. Manufacturers prefer loose tolerances to make parts easier and less expensive to produce. Therefore, tolerance specifications become a critical link between engineering and manufacturing (see Figure 1), a common meeting ground where competing requirements may be resolved.

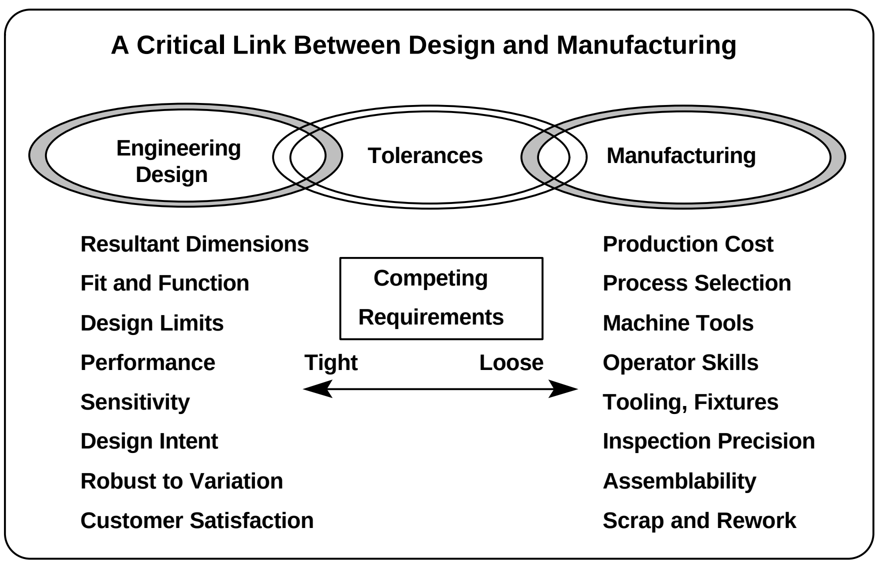

Figure 1. The effects of assigned tolerances are far-reaching

There is a critical need for a rational basis for specifying tolerances. Statistical methods offer powerful analytical tools for predicting the effects of manufacturing variations on design performance and production cost. There are, however, many factors to be considered. Statistical tolerance analysis is a complex problem that must be carefully formulated to assure validity, and then carefully interpreted to accurately determine the overall effect of tolerance assignment on the entire manufacturing enterprise.

Sources of Variation

In order to analyze the effects of the accumulation of component variations on assembled products or mechanical assemblies, all potential sources of variation in an assembly must be included. A comprehensive procedure is presented by which an engineer can systematically create a model for estimating assembly variations for a broad range of product types and applications.

There are three main sources of variation in a mechanical assembly: 1) dimensional variation of individual components, 2) geometric feature variation and 3) variation due to small kinematic adjustments among components which occur at assembly time. The first two are the result of the natural variations in manufacturing processes and the third is a response to process variations. Sources of variation are discussed in detail in section 4.

The two-component assembly shown in Figure 2 demonstrates the relationship between dimensional variations in an assembly and the small kinematic adjustments which occur at assembly time. The assembly has three dimensions that vary, two on the jaw and one on the cylinder, as shown. The variations in the three dimensions have an effect on the distance P. P is important to the function of the assembly and will be referred to as an assembly resultant.

The parts are assembled by inserting the cylinder into the jaw until it makes contact on the two mating surfaces. For each set of parts, the distance P will adjust to accommodate the current value of dimensions A, R, and θ. The assembly resultant P_1 represents the nominal position of the cylinder, while P_2 represents the position of the cylinder when the variations are present. This adjustability of the assembly describes a kinematic constraint, or a closure constraint on the assembly.

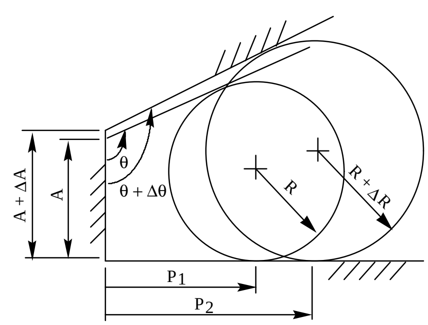

Figure 2. Kinematic adjustment due to component variations

Figure 3 illustrates the same assembly with exaggerated geometric feature variations. For production parts, the contact surfaces are not really flat and the cylinder is not perfectly round. The pattern of surface waviness will differ from one part to the next. In this assembly, the cylinder makes contact on a peak of the lower contact surface, while the next assembly may make contact in a valley. Similarly, the lower surface is in contact with a lobe of the cylinder, while the next assembly may make contact between lobes.

Local surface variations such as these can propagate through an assembly and accumulate just as dimensional variations do. Thus, in a complete assembly model all three sources of variation must be accounted for to assure realistic and accurate results.

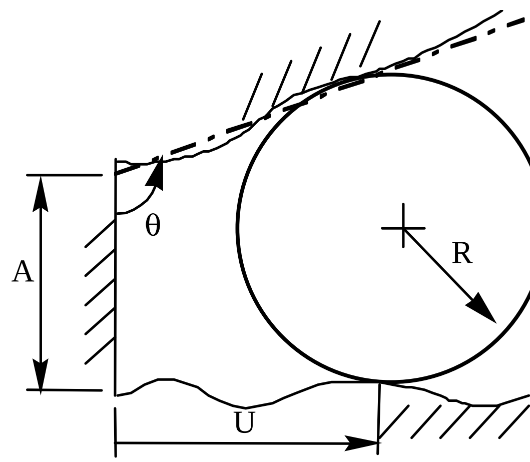

Figure 3. Adjustment due to geometric shape variations

The objective of this paper is to generalize the procedures for computer-aided tolerance modeling and analysis of 3-D mechanical assemblies using vector loop-based assembly models. In vector assembly models, all three variation sources may be included. Of particular interest will be the assembly kinematics employed to set up the kinematic assembly constraints and their solution through the DLM, by which assembly variations can be predicted and evaluated.

This paper consists of a literature review, a description of vector assembly modeling and variation concepts, the DLM formulation for 3-D assemblies and the application to a case study.

2. Research Review

The goal of tolerance analysis is to estimate the variation in assembly parameters from the naturally occurring variations in part dimensions and features. To accomplish this requires the creation of a geometric model of the assembly to which variational analysis may be applied. If process variance data are not available for the part dimensions, the specified tolerances are usually substituted, hence the name "tolerance analysis" is frequently used in connection with this task.

The results of a tolerance analysis are estimates of the mean, variance and other statistical parameters describing the variation of critical assembly features. If engineering design limits have been specified for a feature, quantitative estimates of the percent rejects may be made and compared to desired quality levels.

Five methods have been employed for 2-D and 3-D tolerance analysis: 1) Linearized or Root Sum Square, 2) Method of System Moments, 3) Hasofer-Lind Reliability Index, 4) Direct Integration and 5) Monte Carlo Simulation. These have been discussed in previous papers [Chase & Parkinson 1991, Chase, Gao & Magleby 1995]. All five methods have been primarily limited to problems for which the assembly resultant of interest may be expressed <u>explicitly</u> as a function of the dimensions of the components in an assembly. This raises the question of how to model the assembly.

Establishing explicit assembly functions to describe assembly kinematic adjustments in 3-D space places a heavy burden on the designer. For a general 3-D mechanical assembly, this relationship may be difficult or impossible to obtain. Finding explicit functions for mechanical assemblies also inhibits computer automation of assembly tolerance analysis, since it is very difficult to define such explicit assembly functions in a generalized manner.

An alternative approach to the use of explicit assembly functions is to create a solid model of the assembly on a CAD system. The solid model then serves as the assembly function. Small changes can be simulated and their effects will propagate realistically, provided each part is located relative to its adjacent parts and provided that kinematic adjustments are permitted. This requires that additional constraint capability be added to the solid modeler.

A linearized variation analysis using a solid model as the assembly function has been developed by Turner. To obtain the sensitivities required for calculating assembly variations, the relationship between the parameters defining the model surfaces or solid primitives and the dimensioned quantities appearing on an engineering drawing of the parts must be determined. This was accomplished by making small changes in each of the model variables, calculating the resultant change in the component dimensions and assembly resultants and computing the corresponding sensitivities. The sensitivities were used to form linearized expressions relating the variations in the component dimensions and assembly resultants to variations in the model parameters. Finally, a linear programming problem was set up to find a set of model variations which satisfied the specified component tolerance limits and assembly specifications [Turner and Wozney 1987, 1990, Turner, Wozney and Hoh 1987].

Solid modelers are CPU intensive. Changing a single parameter for a sensitivity calculation requires regeneration of the entire CAD geometry. A detailed model of an assembly may have thousands of model parameters, resulting in a substantial delay for a complete sensitivity calculation on all but the most powerful computers. However, significant progress has been made in reducing the enormous number of sensitivity calculations by prior examination of the model to eliminate non-contributing parameters [Martino and Gabriele 1989]. Feature-based parametric solid modelers should also assist in this reduction process. Additional research efforts by Turner include the addition of kinematic constraints [Turner 1990, Turner and Srikanth 1990, Srikanth and Turner 1990].

Variational geometry is another fundamental approach to assembly modeling. It requires the formulation of analytical equations describing the geometric relationships which must be maintained in an assembly. Constraints such as perpendicular surfaces or surfaces in sliding contact are defined in terms of dimensional parameters. If the design is modified, the system of equations may be solved to adjust the free variables in keeping with the constraints. The advantages of this method are the ease of design iteration and the realistic propagation of manufacturing variations by kinematic adjustments. However, the resulting system of nonlinear equations can become very large and must be solved simultaneously. Also, geometric feature tolerances have yet to be taken into account [Light and Gossard 1982, Gossard et al. 1988, Chung and Schussel 1990].

Lin and Chen [1994] developed a linearized scheme for tolerance analysis of closed loop mechanisms in 3-D space using the 4x4 transformation matrix in the Denavit-Hartenberg (D-H) symbolic notation common to robotics. This method is significant since it has the capability of assessing the influence of each error component on a machine’s accuracy. It is particularly useful to allocate a machine’s tolerances at its design stage and to characterize the performance of an existing machine. Thus far, the method has not been applied to static assemblies. It has only been applied to robotic mechanisms with revolute and prismatic joints (see next section for joint classifications).

The authors' previous paper [Chase, Gao & Magleby 1995] developed the DLM for treating implicit assembly functions, such as kinematic constraints, for assembly tolerance analysis in 2-D space. It is a generalized approach employing vector loops, kinematic joints and geometric feature tolerances. It is a comprehensive system capable of including all three variation sources described earlier.

The DLM reduces the implicit equations describing the kinematic constraints to a set of linear equations for small changes about the nominal, which may be solved directly for the assembly variances. Assembly geometry is reduced dramatically, so it is very efficient computationally. It is suited for integration with commercial CAD systems, by which the required geometry may be extracted directly from a solid model. This paper will extend the DLM method to assemblies in 3-D space.

3. Vector Assembly Models for Tolerance Analysis

In order to create an assembly model for tolerance analysis, specific geometric information and assembly relationships are required. Since an assembly model has usually been created previously in a CAD database, much of this information is available. For current commercial CAD systems, additional information must be added to the solid model. The main elements in this model are datum reference frames, kinematic joints, vectors and vector loops, assembly tolerance specifications, component tolerances and geometric feature tolerances. These elements are, by design, familiar to engineering designers.

Kinematic information is added to the model by creating kinematic joints at mating part interfaces, the type of joint being based on the type of mating contact between parts. The joints are then connected by vectors and linked to form kinematic chains or vector loops. Robison [1989] and Ward [1992] modeled a set of kinematic joint types (Figure 4) to accommodate the possible degrees of freedom in a 3-D assembly. Other constraints, such as geometric feature tolerances and design specifications, must also be added to the vector loops. This procedure is called the assembly tolerance modeling process.

Larsen [1991] further developed Robison's work by automating the procedure of generating the vector loops for assemblies in 2-D space. The algorithms have subsequently been modified for generating vector loop models of assemblies in 3-D space.

In a vector loop-based assembly tolerance model, each vector represents a component dimension. The vectors are arranged in chains or loops representing those dimensions which stack together to determine the resulting assembly dimensions. The design specifications are the engineering limits on those assembly feature variations which are critical to performance.

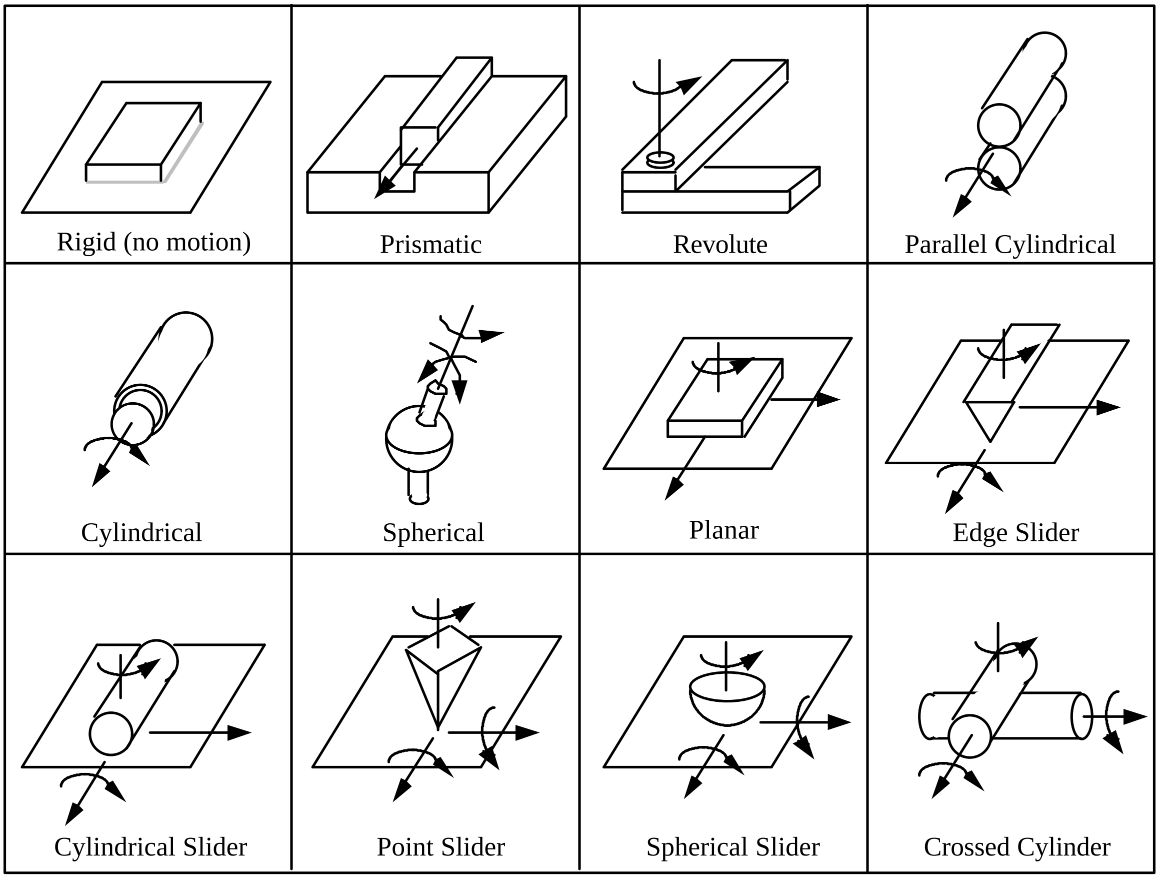

Figure 4. 3-D kinematic joints and their degrees of freedom

Two advantages of vector assembly models over solid assembly models for tolerance analysis are: 1) the geometry is reduced to only those parameters that are required to perform the assembly tolerance analysis, and 2) the derivatives of the assembly function with respect to both the assembly and manufactured dimensions may be obtained more readily from the vector model. This greatly reduces the required computation time. So, assembly variation can be obtained more efficiently, making the system well suited for design iteration and CAD applications.

With a parametric solid model, the changes in the solid model may be automatically reflected in the vector model. This further lends itself to efficient design iteration.

Figure 5 shows a completed vector model overlaid on a 3-D model of a slider crank mechanism. Analysis of this assembly will be presented in a later section.

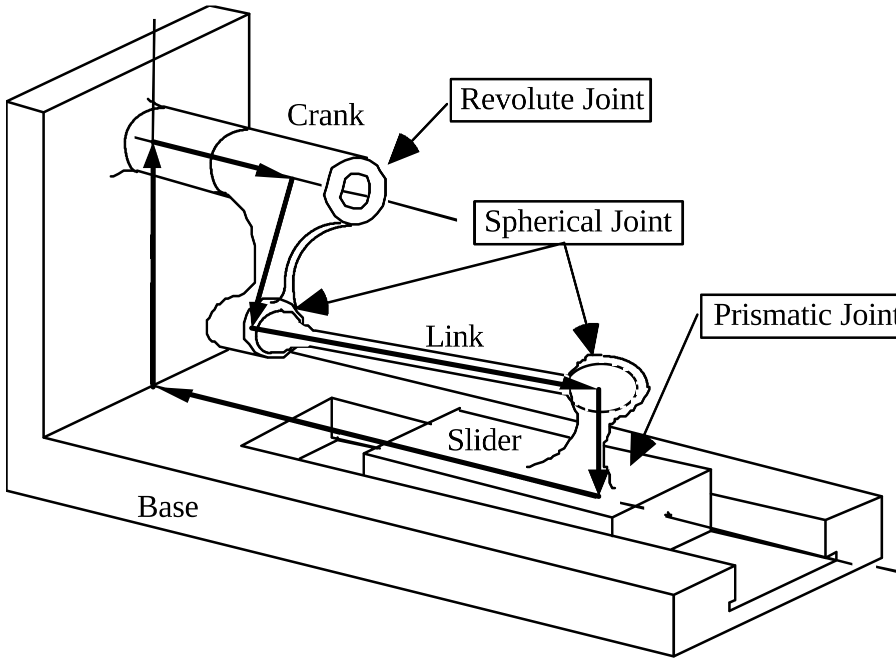

Figure 5. Vector loop model of 3-D crank slider mechanism with kinematic joints

4. Variation Sources in Assemblies

As discussed in the introduction, there are three main variation sources in an assembly. These variations are to be described in greater detail in this section. The three sources form a comprehensive system by which an engineer can systematically create assembly models for variational studies.

Figure 6 shows sample dimensional variations on a component. Such variations are inevitable due to manufacturing process fluctuations, such as tool wear, fixture errors, set up errors, material property variations, temperature, worker skill, etc. The designer must specify size or tolerance limits for each dimension. If the manufactured dimension falls within the specified limits, it is considered acceptable.

Component variations are produced prior to assembly. They are independent, random sources of variation which must be characterized by statistical inspection procedures. Since this variation will affect the performance of the assembled product, it must be carefully controlled.

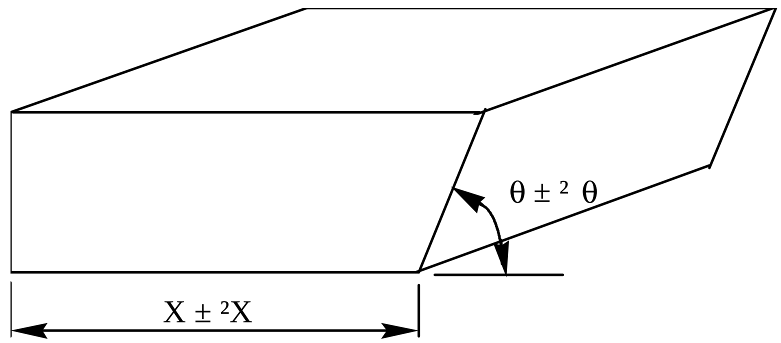

Figure 6. Example of dimensional variations

<u>Geometric feature variations</u> are defined by the ANSI Y14.5M-1982 standard [ASME 1982]. These definitions provide additional tolerance constraints on shape, orientation, and location of produced components. For example, a geometric feature tolerance may be used to limit the flatness of a surface, or the parallelism of one surface on a part relative to established datums, as shown in Figure 7.

In an assembly, geometric feature variations accumulate and propagate similar to dimensional variations. Although generally smaller than dimensional variations, they may be significant in some cases, resulting from rigid body effects. A complete tolerance model of mechanical assemblies should therefore include geometric feature tolerances.

Geometric tolerance limits are established from performance requirements for the final assembly. They contribute to fit and function. They are more difficult to measure and control. Not much data is available. Little has been done to characterize their affect on assemblies. Goodrich [1991] and Ward [1992] have developed models for approximating the effect of geometric variations on mating surfaces. These models describe local variations which may be inserted into vector assembly models and included in statistical tolerance analysis. A more complete discussion and a full catalog of models for various mating surface conditions may be found in the reference by Chase, et al. [1996].

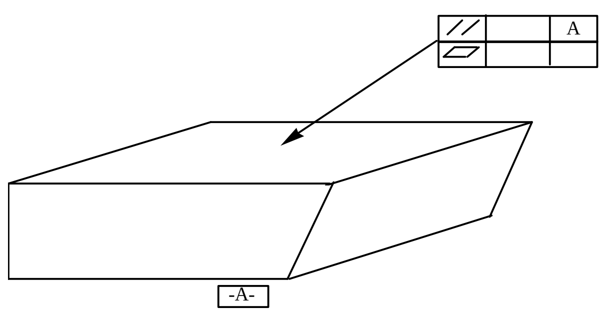

Figure 7. Example of geometric feature variation limits.

Kinematic variations are small adjustments between mating parts which occur at assembly time in response to the dimensional variations and geometric feature variations of the components in an assembly. For example, if the components in the crank slider mechanism (Figure 5) are undersized or oversized, the horizontal position of the slider or the kinematic variable varies around its nominal position, so do the four angular variables at the two ball joints.

Kinematic variations occur at assembly time. Their final dimensions are not known until mating parts are brought together and assembled. The value of a kinematic dimension is dependent on the component dimensions and geometric conditions of the parts selected for assembly. Thus, kinematic variations are dependent variations.

Usually, limiting values of kinematic variations are not specified on the assembly drawing, but critical performance variables, such as a clearance or a location, may appear as assembly specifications. The task for the designer is to assign tolerances to each component in the assembly so that each assembly tolerance specification is met.

The kinematics present in a tolerance analysis model of an assembly is different from the traditional mechanism kinematics. The input and output of the traditional mechanism are large displacements of the corresponding components, such as the rotation of the input and output cranks of a four-bar linkage. In mechanism kinematics, the linkage is composed of rigid bodies, so all the component dimensions remain constant, or fixed at their nominal values.

In contrast to this, the kinematic inputs of an assembly tolerance model are manufacturing process variations, that is, small variations of the component dimensions from their nominal values. The outputs are the small kinematic adjustments between components in response to manufacturing variations, as well as the resulting variation of assembly features, including clearances and fits critical to performance.

The kinematic adjustments in an assembly are not the same as the displacements in a mechanism. They actually represent the difference of each assembly from the ideal or nominal assembly, so they describe the changes from one assembly to the next.

The kinematic assembly equations describe constraints on the interaction between mating component parts. These constraints also serve as functions by which assembly variations may be studied. Since the assembly model is similar to a classical kinematic mechanism model, the analysis principles developed for mechanism kinematics may be applied to assembly tolerance analysis.

It is the kinematic constraints which result in implicit assembly functions, making variational analysis difficult. Current tolerance analysis practices often fail to account for this significant variation source. Kinematic adjustability can significantly alter the statistical accumulation of variance throughout an assembly. If it is not included, variation estimates and predicted rejects will be incorrect.

In a comprehensive assembly tolerance analysis model, all three variations should be included. If any of the three is overlooked or ignored, it can result in significant error. Only when a complete model is constructed, can the designer accurately estimate the variation in the resultant assembly features which are critical to performance.

5. DLM for Mechanical Assemblies in 3-D Space

The DLM is based on the first order Taylor's series expansion of the assembly kinematic constraint equation with respect to both assembly variables and manufactured variables or dimensions of the components in an assembly. Linear algebra is employed to solve the truncated Taylor's series for the variations of the assembly variables in terms of the variations of the manufactured components by either worst case or statistical models for tolerance accumulation.

5.1. Kinematic Constraint Equations

The kinematic constraint for a 3-D mechanical assembly, such as the crank slider mechanism in Figure 5, can be described as a closed vector loop. As the vector loop is traversed from the beginning point to the end point, the dimensions of each component and the rotations and translations of each joint must sum to zero, and the rotations must be such that the coordinate system at the end is congruent with the one at the beginning.

The vector loop equations were derived for 2-D assemblies in the authors' previous paper [Chase, Gao & Magleby 1994]. In that paper, each vector loop equation for 2-D assemblies resulted in three scalar equations, representing the sum of the x and y projections of the dimension vectors in global coordinates and the sum of the relative rotations.

$$H_x = \sum_{i=1}^{n} L_i \cos\left(\sum_{j=1}^{i} f_j\right) = 0 \qquad (1)$$

$$H_y = \sum_{i=1}^{n} L_i \sin\left(\sum_{j=1}^{i} f_j\right) = 0 \qquad (2)$$

$$H_f = \sum_{i=1}^{n} f_j = 0 \text{ or } 360^\circ \qquad (3)$$

where, L_i are the lengths of the dimension vectors and ϕ_j are the rotations from one vector to the next about the z-axis.

In 3-D space, the system of equations is more complicated. The projection equations still hold in global x, y, and z, but the rotation constraints can only be expressed in matrix form.

To express the loop equations in matrix form, each vector may be expressed as a product of transformation matrices. The transformation from joint i-1 to i consists of a combination of three rotation and one translation matrix. We define the convention that the translation is always along the local x axis. For 3-D transformations, we have

$$[R_x] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_x & -\sin\phi_x & 0 \\ 0 & \sin\phi_x & \cos\phi_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4)$$

$$[R_y] = \begin{bmatrix} \cos\phi_y & 0 & \sin\phi_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi_y & 0 & \cos\phi_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

$$[R_z] = \begin{bmatrix} \cos\phi_z & -\sin\phi_z & 0 & 0 \\ \sin\phi_z & \cos\phi_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & L \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

With this convention, the assembly kinematic constraint can be written as equation (8). For a closed loop, the product of all the transformation matrices is equal to the identity matrix.

$$[R_1][T_1][R_2][T_2]...[R_i][T_i]...[R_n][T_n][R_f] = [I] \tag{8}$$

where $[R_i]$: the product of rotation matrices at joint i,
$[T_i]$: the translation matrix at joint i,
$[R_f]$: the final rotation required to bring the loop to closure,
[I]: the identity matrix.

Equation (8) describes a series of rotations and translations to transform the local coordinates from vector-to-vector, until it has traversed the entire vector loop and returned to the starting point. At the tip of each vector, the combined rotations $[R_i]$ serve to align the local x_i coordinate axis with the next vector in the loop. The translation matrix $[T_i]$ contains only one translation component L along the new x axis, corresponding to the length of the next vector in the chain.

From the theory of mechanisms, there are up to six independent equations which can be drawn from equation (8) [Sandor 1984]. Therefore, each loop constraint may be solved for up to six unknowns or assembly variables.

These are nonlinear equations, and require a nonlinear solver. However, for small variations about the nominal of each component in the assembly, the solutions can be approximated by the DLM, a linearized system of equations, for which only the derivatives are needed. Six equations describe the loop variation in the global x, y, z and θ_x, θ_y, θ_z directions. Each has the form:

$$dH_i = \sum_{j=1}^{n} \frac{\partial H_i}{\partial x_j} dx_j + \sum_{k-1}^{m} \frac{\partial H_i}{\partial u_k} du_k \quad \left(i = x, y, z, q_x, q_y, q_z \right) \tag{9}$$

where δx_j are variations in the manufactured dimensions and angles (j = 1...n), δu_k are variations in the dependent assembly variables (k = 1...m), and δH_i is the resultant assembly variation in the corresponding global direction. For closed loops, δH_i is zero and δu_k are the kinematic adjustments required to produce closure.

5.2. Derivative Evaluation

Robison [1989] used a so-called perturbation method for obtaining the derivatives of the assembly constraint equation (8). For a 3-D case, if a translation or rotation at joint i is the variable with respect to which the derivatives are desired, length L is replaced by (L+ ΔL) in the T_i matrix or angle ϕ is replaced by $(\phi + \Delta\phi)$ in the corresponding rotation matrix. The matrix multiplication of equation (8) is then performed, and applied to the zero vector:

For translational variation:

$$[R_1][T_1]...[R_i][T_i(L+\Delta L)]...[R_n][T_n]\{0\ 0\ 0\ 1\}^T = \{\Delta X\ \Delta Y\ \Delta Z\ 1\}^T \quad (10)$$

For rotational variation:

$$[R_1][T_1]...[R_i(\phi + \Delta\phi)][T_i]...[R_n][T_n]\{0\ 0\ 0\ 1\}^T = \{\Delta X\ \Delta Y\ \Delta Z\ 1\}^T \quad (11)$$

Due to the small perturbation, the loop equation no longer closes, resulting in a closure error vector $\{\Delta X\ \Delta Y\ \Delta Z\ 1\}^T$.

The derivatives may then be approximated numerically by varying one length or angle dimension at a time.

$$\begin{array}{ll}
\text{Translational variable} & \text{Rotational variable} \\
\dfrac{\partial H_x}{\partial L} \cong \dfrac{\Delta X}{\Delta L} & \dfrac{\partial H_x}{\partial f} \cong \dfrac{\Delta X}{\Delta f} \\
\dfrac{\partial H_y}{\partial L} \cong \dfrac{\Delta Y}{\Delta L} & \dfrac{\partial H_y}{\partial f} \cong \dfrac{\Delta Y}{\Delta f} \\
\dfrac{\partial H_z}{\partial L} \cong \dfrac{\Delta Z}{\Delta L} & \dfrac{\partial H_z}{\partial f} \cong \dfrac{\Delta Z}{\Delta f} \\
\dfrac{\partial H_{qx}}{\partial L} = 0 & \dfrac{\partial H_{qx}}{\partial f} = \cos a \\
\dfrac{\partial H_{qy}}{\partial L} = 0 & \dfrac{\partial H_{qy}}{\partial f} = \cos b
\end{array} \quad (12)$$

$$\frac{\partial H_{qz}}{\partial L} = 0 \qquad\qquad \frac{\partial H_{qz}}{\partial f} = \cos g$$

where H_x, H_y and H_z are the translational constraints, and $H_{\theta x}$, $H_{\theta y}$ and $H_{\theta z}$ are the rotational constraints in x, y and z direction respectively, and α, β and γ are the global direction cosine angles of the local axis around which the rotation is made [Gao 1993].

5.3. Linearization of the Implicit Assembly Constraints

The first order Taylor's series expansion of the closed loop kinematic constraints, equation (9), can be written in matrix form:

$$\{\delta H\} = [A]\{\delta X\} + [B]\{\delta U\} = \{\Theta\} \qquad (13)$$

where {δH}: vector of the clearance variations,
{δX}: vector of the variations of the manufactured variables,
{δU}: vector of the variations of the assembly variables,
[A]: matrix of the first order partial derivatives of the manufactured variables,
[B]: matrix of the first order partial derivatives of the assembly variables.
{Θ}: the zero vector.

Matrices [A] and [B] can be obtained by the method discussed above. Each column of the [A] matrix takes following the format,

$$\{A_i\} = \left\{ \frac{\partial H_x}{\partial x_i}, \frac{\partial H_y}{\partial x_i}, \frac{\partial H_z}{\partial x_i}, \frac{\partial H_{qx}}{\partial x_i}, \frac{\partial H_{qy}}{\partial x_i}, \frac{\partial H_{qz}}{\partial x_i} \right\}^t \qquad (14)$$

where x_i is the ith manufactured dimension. Matrix [B] has the same column notation except that the variable is u_i instead of x_i.

To correctly map the derivatives into the A or B matrices requires that each vector and rotation in the loop be identified as either a dependent or independent variable or a constant. A set of modeling rules is required when creating the model, which assure the proper relationships between the vectors passing through each joint and the joint axes. Consistent relationships permit construction of algorithms for correctly making the determination and performing the mapping [Chase & Trego 1994].

Solving equation (13) for ΔU gives (assuming that [B] is a full-ranked matrix):

$$\{\delta U\} = -[B]^{-1}[A]\{\delta X\} \qquad (15)$$

In equation (15), matrix [A] describes the geometric sensitivity to component variations δX and matrix $[B]^{-1}$ imposes the adjustments along the correct kinematic joint axes to achieve closure. The significance of this equation is that the assembly variations may be obtained directly from the geometry by simple matrix algebra operations. And, once the matrices have been obtained, trial values of δX may be applied without repeating the solution.

If the [B] matrix is singular, that means the system is over-determined, and a least square fit must be applied to solve equation (12).

$$\{\delta U\} = -([B]^T[B])^{-1}[B]^T[A]\{\delta X\} \qquad (16)$$

For an <u>open loop</u> kinematic constraint, there may also be one or more closed loop kinematic constraints which the assembly must satisfy. The strategy for such a system of assembly constraints is to solve the closed loop constraints first, and then substitute the solution in the open loop kinematic constraint. Finally, evaluate the variations of the open loop variables.

$$\{\delta V\} = [C]\{\delta X\} + [D]\{\delta U\} \qquad (17)$$

where δV: the variations of the open loop assembly variables,
[C]: the first order derivative matrix of the manufactured variables in the open loop, [D]: the first order derivative matrix of the assembly variables in the open loop.

If [B] is full-ranked, equation (17) can be written as:

$$\{\delta V\} = ([C] - [D][B]^{-1}[A])\{\delta X\} \qquad (18)$$

or, when [B] is not,

$$\{\delta V\} = ([C] - [D]([B]^T[B])^{-1}[B]^T[A])\{\delta X\} \qquad (19)$$

5.4. Estimation of Kinematic Variations and Assembly Rejects

The estimation of the kinematic variations in an assembly can be obtained from equation (15) or (16) for the closed loop constraint, or equation (18) or (19) for the open loop constraint, by a worst case or statistical model.

Worst case:

$$dU_i = \sum_{j=1}^{n} |S_{ij}|\, tol_j \cong T_{ASM_i} \qquad (i = 1,\ldots,m) \tag{20}$$

Statistical model:

$$dU_i = \sqrt{\sum_{j=1}^{n} \left(S_{ij}\, tol_j\right)^2} \cong T_{ASM_i} \qquad (i = 1,\ldots,m) \tag{21}$$

where m is the number of assembly variables which are critical to the design, n is the number of contributing manufactured dimensions, tol_j is the tolerance of the jth manufactured dimension, $T_{ASM\ i}$ is the design specification for the ith assembly variable U_i and S_{ij} are elements of the sensitivity matrix [S] of the assembly constraint. In determined and over-determined systems, [S] is given respectively by:

For closed loop constraints

$[S] = -[B]^{-1}[A]$ (determined) (22)

$[S] = -([B]^T[B])^{-1}[B]^T[A]$ (over-determined) (23)

For open loop constraints

$[S] = [C] - [D][B]^{-1}[A]$ (determined) (24)

$[S] = [C] - [D]([B]^T[B])^{-1}[B]^T[A]$ (over-determined) (25)

The estimation of the assembly rejects is based on the assumption that the distribution of the kinematic variable(s) is Normal, which is a reasonable estimate for most assemblies. The estimate of a kinematic variation is treated as representing three standard deviations, and this deviation together with the mean value of the kinematic variable may be used to estimate by either integration or table the assembly rejects for a given assembly batch.

6. Example

As an example to demonstrate the procedure of applying DLM to a real assembly, the 3-D crank slider mechanism is re-examined in detail. The assembly consists of a base, a crank, a link and a slider. The crank rotates around its shaft. This rotation is transmitted by the crank lever to the connecting rod or link. The connecting rod pivots around the “ball joints” as the slider moves forward and backward. The dimensions which govern the operation of the crank slider assembly are shown in Figure 8.

The assembly constraint equation for this mechanism can be obtained by rotating and translating the local joint coordinate system around the assembly vector loop. When the vector loop is traversed, the assembly constraint equation in the form of equation (8) results. The dependent kinematic variables must be identified before the system can be analyzed. Vectors A, B, C, D and E represent manufactured dimensions, while vector U and rotations ϕ_1, ϕ_2, ϕ_3, and ϕ_4 are kinematic assembly variables. For measured variations in the manufactured dimensions A through E, we desire an estimate of the resulting variation in assembly variables U through ϕ_4.

For analysis, the input crank is rotated to some predetermined position, therefore, it is not an unknown. The two spherical joints are defined with two rotational degrees of freedom [Sandor 1984]. Together, they have four degrees of freedoms. The slider can move only forward and backward, so it has one degree of freedom. The rotation of the connecting rod around its axis is arbitrary and therefore meaningless to the tolerance analysis. So, the total kinematic or assembly variables in this system are five, instead the six possible for a 3-D assembly. The resulting system of equations will be over-determined.

The geometric data of the crank slider mechanism is given in Table 1, with the global reference frame defined in Figure 8.

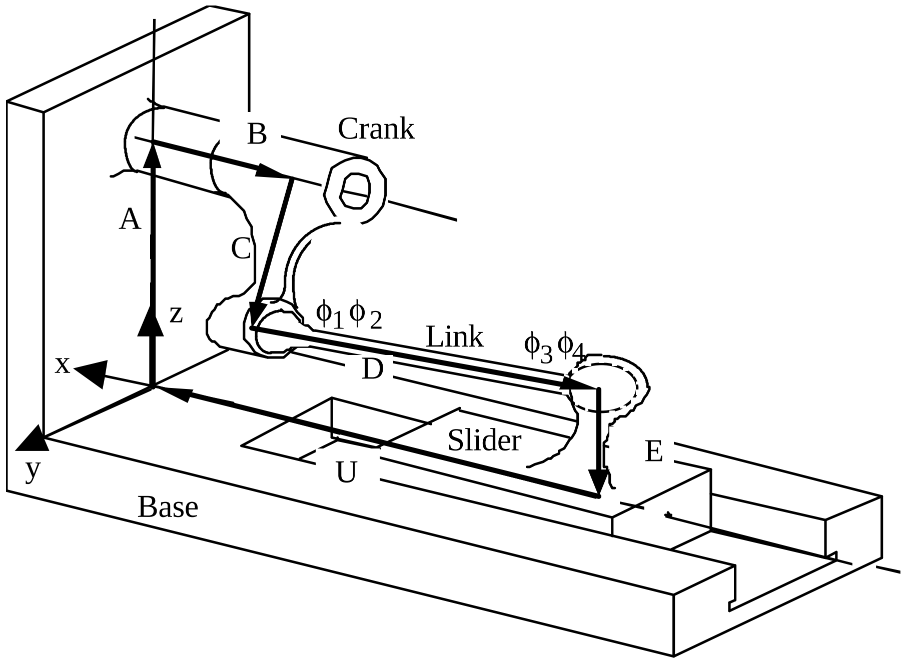

Figure 8. 3-D crank slider mechanism with dimensions

Table 1 Geometric data of the crank slider mechanism

Part Name	Transformation	Nominal Dimension	Tolerance(±)
Height of base	Rotation	y = -90° z = 0°	
	Translation **A**	20	0.025
Position of crank	Rotation	y = -90° z = 0°	
	Translation **B**	12	0.0125
Length of crank arm	Rotation	y = -90° z = 45°	
	Translation **C**	15	0.0125
Length of link	Rotation ϕ_1 ϕ_2	y=99.007° z=-20.705°	? ?
	Translation **D**	30	0.03
Height of slider	Rotation ϕ_3 ϕ_4	y=-78.157° z=-44.473°	? ?
	Translation **E**	5	0.0025
Position of slider	Rotation	y = -90° z = -29.298°	
	Translation **U**	39.7164	?

The derivative matrix with respect to the assembly variables can be expressed as:

$$[B]=\begin{bmatrix} \frac{\partial H_x}{\partial f_1} & \frac{\partial H_x}{\partial f_2} & \frac{\partial H_x}{\partial f_3} & \frac{\partial H_x}{\partial f_4} & \frac{\partial H_x}{\partial U} \\ \frac{\partial H_y}{\partial f_1} & \frac{\partial H_y}{\partial f_2} & \frac{\partial H_y}{\partial f_3} & \frac{\partial H_y}{\partial f_4} & \frac{\partial H_y}{\partial U} \\ \frac{\partial H_z}{\partial f_1} & \frac{\partial H_z}{\partial f_2} & \frac{\partial H_z}{\partial f_3} & \frac{\partial H_z}{\partial f_4} & \frac{\partial H_z}{\partial U} \\ \frac{\partial H_{qx}}{\partial f_1} & \frac{\partial H_{qx}}{\partial f_2} & \frac{\partial H_{qx}}{\partial f_3} & \frac{\partial H_{qx}}{\partial f_4} & \frac{\partial H_{qx}}{\partial U} \\ \frac{\partial H_{qy}}{\partial f_1} & \frac{\partial H_{qy}}{\partial f_2} & \frac{\partial H_{qy}}{\partial f_3} & \frac{\partial H_{qy}}{\partial f_4} & \frac{\partial H_{qy}}{\partial U} \\ \frac{\partial H_{qz}}{\partial f_1} & \frac{\partial H_{qz}}{\partial f_2} & \frac{\partial H_{qz}}{\partial f_3} & \frac{\partial H_{qz}}{\partial f_4} & \frac{\partial H_{qz}}{\partial U} \end{bmatrix}$$

$$=\begin{bmatrix} 0.8579 & -13.967 & -3.1115 & -2.4467 & 1 \\ 8.4857 & -9.8516 & 26.078 & 4.3604 & 0 \\ -8.4857 & -6.7201 & -24.716 & -19.436 & 0 \\ 0 & -0.1566 & -0.3492 & 0.8721 & 0 \\ 0.7071 & 0.6984 & 0.6223 & 0.4894 & 0 \\ 0.7071 & -0.6984 & 0.7006 & 0 & 0 \end{bmatrix} \qquad (26)$$

The derivative matrix with respect to the manufactured variables can be obtained similarly.

$$[A]=\begin{bmatrix} \frac{\partial H_x}{\partial A} & \frac{\partial H_x}{\partial B} & \frac{\partial H_x}{\partial C} & \frac{\partial H_x}{\partial D} & \frac{\partial H_x}{\partial E} \\ \frac{\partial H_y}{\partial A} & \frac{\partial H_y}{\partial B} & \frac{\partial H_y}{\partial C} & \frac{\partial H_y}{\partial D} & \frac{\partial H_y}{\partial E} \\ \frac{\partial H_z}{\partial A} & \frac{\partial H_z}{\partial B} & \frac{\partial H_z}{\partial C} & \frac{\partial H_z}{\partial D} & \frac{\partial H_z}{\partial E} \\ \frac{\partial H_{qx}}{\partial A} & \frac{\partial H_{qx}}{\partial B} & \frac{\partial H_{qx}}{\partial C} & \frac{\partial H_{qx}}{\partial D} & \frac{\partial H_{qx}}{\partial E} \\ \frac{\partial H_{qy}}{\partial A} & \frac{\partial H_{qy}}{\partial B} & \frac{\partial H_{qy}}{\partial C} & \frac{\partial H_{qy}}{\partial D} & \frac{\partial H_{qy}}{\partial E} \\ \frac{\partial H_{qz}}{\partial A} & \frac{\partial H_{qz}}{\partial B} & \frac{\partial H_{qz}}{\partial C} & \frac{\partial H_{qz}}{\partial D} & \frac{\partial H_{qz}}{\partial E} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & -0.9239 & 0 \\ 0 & 0 & 0.7071 & -0.3535 & 0 \\ 1 & 0 & -0.7071 & -0.1464 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (27)$$

Equation (22) can not be used to find the sensitivity matrix since the [B] matrix is not a square matrix. Therefore, equation (23) must be used to find a least square fit solution.

$$[\,S\,] = -([\,B\,]^T[\,B\,])^{-1}[\,B\,]^T[\,A\,]$$

$$= \begin{bmatrix} -0.0290 & 0 & 0.0355 & -0.0032 & 0.0290 \\ 0.0308 & 0 & 0.0064 & -0.0186 & -0.0308 \\ 0.0156 & 0 & -0.0345 & 0.0095 & -0.0156 \\ 0.0336 & 0 & -0.0102 & -0.0117 & -0.0336 \\ 0.5860 & 1 & -0.0735 & 0.6677 & -0.5860 \end{bmatrix} \qquad (28)$$

With the sensitivity matrix known, the variations of the kinematic or assembly variables can then be calculated by applying either equation (20) or (21).

Worst case: Statistical model:

$$\begin{Bmatrix} \delta\phi_1 \\ \delta\phi_2 \\ \delta\phi_3 \\ \delta\phi_4 \\ \delta U \end{Bmatrix} = \begin{Bmatrix} 0.0766^\circ \\ 0.0851^\circ \\ 0.0656^\circ \\ 0.0804^\circ \\ 0.0496 \end{Bmatrix} \qquad \begin{Bmatrix} \delta\phi_1 \\ \delta\phi_2 \\ \delta\phi_3 \\ \delta\phi_4 \\ \delta U \end{Bmatrix} = \begin{Bmatrix} 0.0492^\circ \\ 0.0549^\circ \\ 0.0371^\circ \\ 0.0528^\circ \\ 0.0278 \end{Bmatrix} \qquad (29)$$

In this assembly, dimension U has specified design limits, since its value and variation will affect the performance of the mechanism. If the design spec for U is set, and the estimated variation δU equals 3.0 standard deviations, then the assembly reject rate can be calculated by either standard normal distribution table or integration or empirical methods.

This example is evaluated at the predetermined position of the crank arm. However, this procedure can be applied repeatedly to find the maximum variation of the position of the slider as the crank arm is incremented through 360 degrees.

7. Conclusions

This paper has presented a comprehensive method for modeling and analyzing variation in 3-D mechanical assemblies. It will make possible new CAD tools for engineering designers which integrate manufacturing considerations into the design process. Using this method, designers will be able to quantitatively predict the effects of variation on performance and producibility.

After the product is in production, manufacturing systems personnel can substitute more accurate data into the model. Tolerances may be reallocated among the components to reduce overall cost. "What-if" studies can be performed to determine the effect of vendor-supplied components which are out of spec.

If design and manufacturing personnel can adopt a common engineering model for assemblies, it can serve as a vehicle for resolving their often competing tolerance requirements. Tolerance analysis can become a common meeting ground where they can work together to systematically pursue cost reduction and quality improvement.

The 3-D DLM approach described in this paper is compatible with product design and development processes. It is well suited to design iteration and optimization. It is also suited to the technical skills of engineering designers. Modeling elements are familiar to most engineers. A high level of integration with commercial CAD systems should enhance its acceptance by the design community. Testing of the approach is now in progress to assure accuracy and validity. A preliminary study compared the DLM assembly tolerance analysis method with Monte Carlo simulation [Gao 1993, Gao, et al. 1995]. The results show that the DLM produces accurate estimates of variation for a broad range of assembly applications.

The geometrical feature variations of the components in an assembly discussed in the introduction and variation sources sections were not included in this paper. A detailed treatment will be discussed in a separate paper in the future.

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References

ASME, 1982, "Dimensioning and Tolerancing," ANSI Y14.5M - 1982, American Society of Mechanical Engineers, New York, NY.

Chase, K. W., Gao, J., and Magleby, 1995, S. P., "General 2-D Tolerance Analysis of Mechanical Assemblies with Small Kinematic Adjustments," J. of Design and Manufacture, v 5, pp. 263-274.

Chase, K. W., J. Gao, S. P. Magleby and C. D. Sorenson, 1996, "Including Geometric Feature Variations in Tolerance Analysis of Mechanical Assemblies", IIE Transactions, v 28, pp. 795-807.

Chase, K. W., and Trego, A., 1994, "AutoCATS--Computer-Aided Tolerancing System, 2-D Modeling User Guide," ADCATS publication, Brigham Young University, Provo, UT.

Chase, K. W., and Parkinson, A.R., 1991, "A Survey of Research in the Application of Tolerance Analysis to the Design of Mechanical Assemblies," Research in Engineering Design, v 3, pp. 23-37.

Chung, J. C. H., and Schussel, M. D., 1990, "Technical Evaluation of Variational and Parametric Design," Computers in Engineering-1990, v 1, ASME, pp. 289-98.

Gao, J., 1993, "Nonlinear Tolerance Analysis of Mechanical Assemblies," Dissertation, Mechanical Engineering Department, Brigham Young University, Provo, UT.

Gao, J., Chase, K. W., and S. P. Magleby, "Comparison of Assembly Tolerance Analysis by the Direct Linearization and Modified Monte Carlo Simulation Methods," Proc. of the ASME Design Engineering Technical Conferences, Boston, MA, 1995, pp. 353-360.

Goodrich, C. G., 1991, "Representation and Modeling of Geometric Form Variations for 3-D Tolerance Analysis," M.S. Thesis, Mechanical Engineering Department, Brigham Young University, Provo, UT.

Gossard, D. C., Zuffante, R. P., and Sakurai, H., 1988, "Representing Dimensions, Tolerances, and Features in MCAE Systems," IEEE Computer Graphics & Applications, v 8 n 2, pp. 51-59, March.

Larsen, G. C., 1991, "A Generalized Approach to Kinematic Modeling for Tolerance Analysis of Mechanical Assemblies," M.S. Thesis, Mechanical Engineering Department, Brigham Young University, Provo, UT.

Light, R., and Gossard, D. C., 1982, "Modification of Geometric Models through Variational Geometry," Computer-Aided Design, v14 n4, pp. 209-214, July.

Lin, P. D., and Chen, J. F., 1994, "Analysis of Errors in Precision for Closed Loop Mechanisms," Journal of Mechanical Design, vol 116, pp. 197-203.

Martino, P., Gabriele, G. A., 1989, “Application of Variational Geometry to the Analysis of Mechanical Tolerances,” Failure Prevention and Reliability - 1989, ASME Paper No. DE-Vol 16, pp 19.-27.

Robison, R. H., 1989, “A Practical Method for Three-Dimensional Tolerance Analysis Using a Solid Modeler,” M.S. Thesis, Mechanical Engineering Department, Brigham Young University, Provo, UT.

Sandor, G. N., Erdman, A. G., 1984, “Advanced Mechanism Design: Analysis and Synthesis, Volume 2,” Prentice-Hall, INC.

Srikanth, S., Turner, J., 1990, “Toward a Unified Representation of Mechanical Assemblies,” Engineering with Computers, vol 6, pp. 103-12.

Turner, J., 1990, “Relative Positioning of Parts in Assemblies Using Mathematical Programming,” Computer-Aided Design, v 22 n 7, pp. 394-400, Sept.

Turner, J., and Srikanth, S., 1990, “Constraint Representation and Reduction in Assembly Modeling and Analysis,” Rensselear Design Research Center, Tech Report No. 90027.

Turner, J., Wozny, M., 1987, “Tolerances in Computer-Aided Design,” The Visual Computer, n 3, pp. 214-226.

Turner, J., Wozny, M., 1990, “The M-Space Theory of Tolerances,” Advances in Design Automation-1990, vol 1, ASME Publication No. DE- vol 23-1, pp. 217-225.

Turner, J., Wozny, M., Hoh, D., 1987, “Tolerance Analysis in a Solid Modeling Environment,” Computers in Engineering-1987, vol 2, ASME, pp. 169-175.

Ward, K., 1992, “Integrating Geometric Form Variations into Tolerance Analysis of 3D Assemblies,” M. S. Thesis, Mechanical Engineering Department, Brigham Young University, Provo, UT.