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Comparison of Assembly Tolerance Analysis by the Direct Linearization and Modified Monte Carlo Simulation Methods

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ABSTRACT

Two methods for performing statistical tolerance analysis of mechanical assemblies are compared: the Direct Linearization Method (DLM), and Monte Carlo simulation. A selection of 2-D and 3-D vector models of assemblies were analyzed, including problems with closed loop assembly constraints. Closed vector loops describe the small kinematic adjustments that occur at assembly time. Open loops describe critical clearances or other assembly features. The DLM uses linearized assembly constraints and matrix algebra to estimate the variations of the assembly or kinematic variables, and to predict assembly rejects. A modified Monte Carlo simulation, employing an iterative technique for closed loop assemblies, was applied to the same problem set. The results of the comparison show that the DLM is accurate if the tolerances are relatively small compared to the nominal dimensions of the components, and the assembly functions are not highly nonlinear. Sample size is shown to have great influence on the accuracy of Monte Carlo simulation.

INTRODUCTION

The linearization method and Monte Carlo simulation are the most commonly used methods for statistical tolerance analysis of mechanical assemblies, due to the versatility and speed of the linearization method and the nonlinear capability and accuracy of Monte Carlo simulation. The concern for using the linearization method is its accuracy, and so far, little information on accuracy is available to analysts. For Monte Carlo simulation, accuracy is related to the sample size, although the relationship of sample size and accuracy has not been well defined.

Traditionally, both the linearization method and Monte Carlo simulation for statistical tolerance analysis of mechanical assemblies are applied to explicit assembly functions, that is, the assembly feature or dimension must be expressed in terms of the component dimensions in the assembly (Cox 1986, Shapiro & Gross 1981, DeDoncker & Spencer 1987, Doepker & Nies 1989, Early & Thompson 1989, Fuscaldo 1991). In 2-D or 3-D space, this function is usually a nonlinear implicit function of the assembly variables. It is very difficult or impossible for a designer to establish an explicit function for “real world” assemblies. The authors have developed a generalized linearization method and modified Monte Carlo simulation for 2-D and 3-D assembly tolerance analysis using implicit assembly functions (Chase, Gao & Magleby 1995a, Gao 1993). The linearization method is called the Direct Linearization Method (DLM). These two methods will be described.

This paper applies the DLM and Monte Carlo simulation using implicit assembly functions to a set of mechanical assemblies in both 2-D and 3-D space. The results from the two methods are compared. A Monte Carlo simulation with a very large sample size is chosen as the “exact” solution. The effect of sample size on the accuracy of the Monte Carlo simulation is also investigated.

The DLM and modified Monte Carlo simulation using an implicit assembly function, were developed to take advantage of the increased use of CAD in product design. Solid modelers are used to create assembly models, and an assembly tolerance modeler is employed to include the tolerances of the components in the assembly models. Assembly tolerance analysis and allocation can then proceed on the assembly models using the DLM and Monte Carlo simulation. This section will discuss the vector-loop-based assembly tolerance modeler, and both the DLM and modified Monte Carlo simulation methods for assembly tolerance analysis.

Vector-Loop-based Assembly Tolerance Modeler

The DLM and Monte Carlo simulation were applied to the same vector loop based assembly tolerance models (Chase, Gao & Magleby 1995a, Chase, Gao & Magleby 1995b). Solid modelers do not generally contain the controlled dimension and tolerance data that are required for tolerance analysis. The vector loop assembly tolerance model extends the functions and data structure of the solid modeler to include tolerancing capabilities. It allows the designer to create 2-D and 3-D vector assembly tolerance models graphically and add them to the solid model as objects. The vector model is stored as part of the solid model database. The model contains the complete dimension and tolerance information required for performing tolerance analysis. The complete model may then be accessed by the tolerance analysis module which will perform statistical tolerance analysis and tolerance allocation (CATS Modeler 1994).

Manufactured parts are seldom used as single parts. They are used in assemblies of parts. The dimensional variations which occur in each component part of an assembly accumulate statistically and propagate kinematically, causing the overall assembly dimensions to vary according to the number of contributing sources of variation. The resultant critical clearances and fits which affect performance are thus subject to variation due to the tolerance stackup of the component part variations.

The three major sources of variation in assemblies are included in the models:

1) dimensional (lengths and angles)
2) geometric feature (ANSI Y 14.5)
3) kinematic (small internal adjustments)

The model is based on a graphically generated vector chain(s) or loop(s) representing a mechanical assembly. Each vector represents a component dimension. Complex assemblies may require solving several vector loops simultaneously. Contact between mating parts is described by kinematic joints, (planar, slider, pin joints, etc.), which assist the designer to conceptually understand the adjustability within the assembly. Kinematic constraints assure that variations propagate through the assembly in a realistic way. Figure 1 shows the kinematic joints, datums and vector loop for the one-way clutch assembly discussed in a previous paper (Chase, Gao & Magleby 1995a).

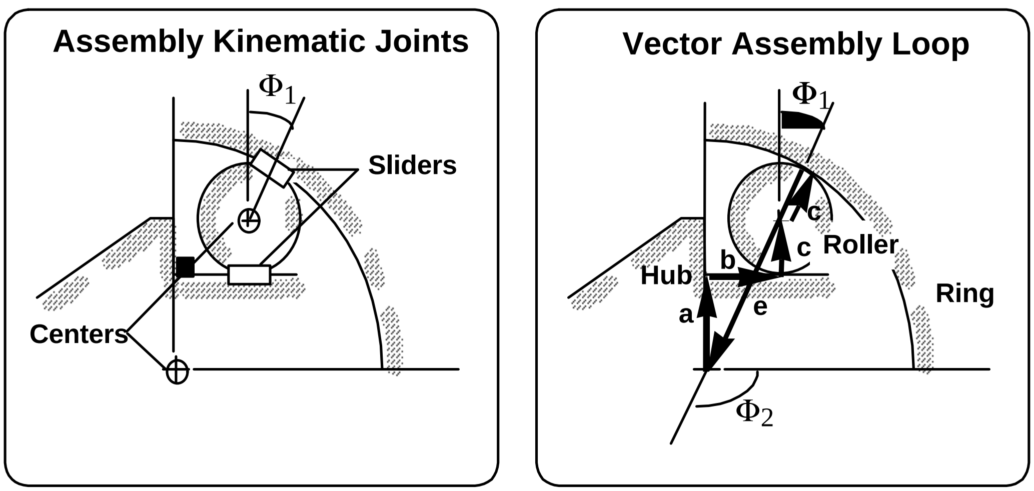

FIGURE 1. ASSEMBLY KINEMATIC JOINTS, DATUMS AND VECTOR LOOP OF THE ONE-WAY CLUTCH.

In the one-way clutch assembly, the manufactured dimensions are a, c and e. The dimensions b, ϕ_1 and $_2$ are the assembly or

manufactured dimensions. The assembly or kinematic variables must be able to adjust to accommodate the variations of the

Shigley 1969). ϕ is the critical

the clutch. The goal of assembly tolerance analysis of the clutch is to find out the effect of the tolerances of a, c and e on the

ϕ_1

The effects of ANSI Y 14.5 geometric feature control tolerances can also be included in the model. Models for propagating the corresponding kinematic joint types. With different joint types and 14 geometric feature controls, there are

relationships to define, but these are already tabulated (Chase, Gao &

The assembly constraint for the one-way clutch can be expressed in the implicit function,

$$[R_a \quad _a][R \ ][T_b \quad _{c1}][T \quad][R_{c2} \quad _{c2}][R \ ][T_e \quad _f] = [I]$$

where R represents the rotational transformation and T represents the $_f]$

rotation to make the vector loop close.

so the designer can express his design intent by placing tolerance limits on critical assembly features. Such features as the gap, parallelism and flushness of a car door and its mating door frame vary as the result of tolerance accumulation of a chain of component dimensions. By analyzing the variation in the dimensional chain, the gap variation may be estimated and assembly rejects predicted.

Direct Linearization Method

The DLM quantitatively estimates the variation of all critical assembly features and predicts the percent of assemblies which will fail to meet the design specs. It uses the kinematic assembly constraints as its assembly functions. For closed vector loops, the starting and ending reference frame must be located at the same point and oriented in the same direction when every vector in the loop has been traversed (equation (1)). These constraints result in nonlinear implicit functions of the assembly variables, which are difficult to solve.

The assembly constraint equations may be linearized by a first order Taylor's series expansion and solved by linear algebra for the assembly variations in terms of component tolerances in the assembly. The assembly variations can then be obtained statistically and the assembly rejects can be predicted with the assembly variation for a given design specification (Chase, Gao & Magleby 1995a).

The DLM can also be used for tolerance allocation with predefined schemes, such as proportional scaling. Allocation is a tolerance selection tool to assist the designer in distributing the available assembly tolerance among the components of an assembly. Tolerances may be loosened on expensive processes and tightened on others to reduce cost while assuring that the design specs will be met.

Modified Monte Carlo Simulation

analysis using a random number generator which selects values

distribution assigned by the designer. These values are combined through the assembly function to determine a series of

first four moments of the assembly variable, and finally, the moments can be used to determine the behavior of the assembly, which fall outside the design specifications, or assembly reject rate. For each simulation, the value of the assembly variables

Carlo Simulations will be accurate, provided careful attention has been given to proper sample size and accurate generation of

Since the assembly constraint equations are nonlinear implicit functions, an iterative solution must be obtained for each

the complexity of the assembly constraints. For 3-D assemblies, there may be more equations than the number of unknowns. A

The procedures for the Monte Carlo simulation using implicit functions can be outlined as:

Generate random variates representing each of the

The variations are drawn from the specified statistical distributions. The nominal dimensions of the

serve as the input to the nonlinear assembly system.

2.

constraints act as the assembly functions describing the unknown assembly or kinematic variables. These

3. Choose the appropriate nonlinear solvers and solve for

solution).

4.

simulated assemblies to generate a statistical distribution for the assembly features of interest.

Apply limits to those features of interest. Use statistics to evaluate the simulated results of the

The procedure is illustrated graphically in Figure 2.

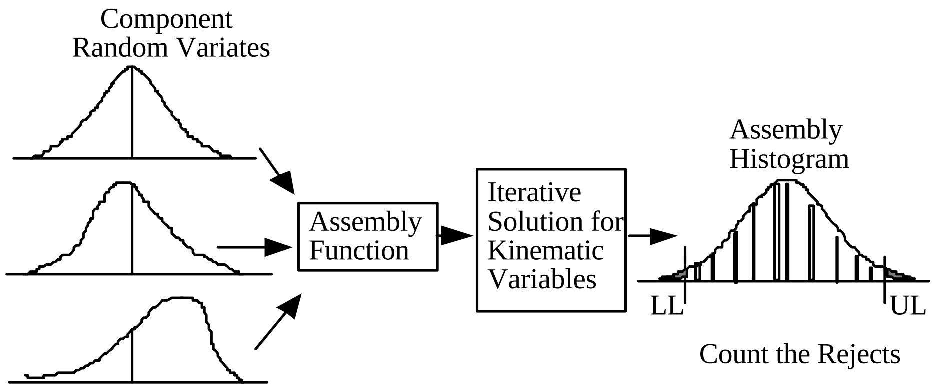

FIGURE 2. MONTE CARLO SIMULATION FOR IMPLICIT ASSEMBLY CONSTRAINTS

TEST CASES

Seven 2-D assemblies and one 3-D assembly are included in the case studies. The complexity of each assembly is a function of the number of parts and the nature of the contact between parts. Sliding contact or rotational adjustments between mating parts are described by inserting kinematic joints into the vector model at points of contact. Each kinematic joint introduces degrees of freedom or variable dimensions which are the unknowns in the assembly equations. A closed vector loop will have three unknown kinematic variables, two closed loops will have six, etc., in 2-D space. Simultaneous solution of the assembly equations is required to determine the unknown variations. A summary of the size and complexity of the assembly modeling of these examples is reflected in Table 1. Complete dimensions, descriptions and sources of the example problems are presented elsewhere (Gao 1993).

TABLE 1. COMPLEXITY OF EXAMPLE ASSEMBLY MODELS

Assembly	#Parts	#Joints	#D_vector	#F_vector	#C_loops	#O_loops	#Unkowns
Clutch	3	5	5	4	1	0	3
Bike	3	9	9		1	0	3
Block	3	10	12		3	0	9
Ratchet	3	9	8		2	0	6
Tape Reel	4	10	10	4	1	1	4
Positioner	6	7	9		2	1	9
Quick slider	4	6	5		2	1	7
3D crank	5	6	6		1	0	5

Where # = the number of
D_vector = dimension vector
F_vector = geometric feature variables
C_loops = closed loops
O_loops = open loops

COMPARISON OF RESULTS

In this section, the results of the linearized analysis or the DLM solutions will be compared to Monte Carlo simulation. A Monte Carlo simulation with a sample size of 100,000 assemblies has been established as the reference value for each sample problem. Also investigated is the effect of sample size on the accuracy of Monte Carlo estimates of the assembly variations and the predicted rejects for the assemblies.

The DLM vs. Monte Carlo Simulation

The computed standard deviation of a critical assembly feature for 10 test problems is presented in Table 2. The assembly variation is the result of tolerance stackup of the contributing component dimensions in each assembly. In Table 2, the * sign means that the result includes both dimensional and geometrical feature tolerances of the components in the assembly.

TABLE 2. SUMMARY OF VARIATION RESULTS - DLM VS. SIMULATION

Assembly Name	Variation of Controlled Assembly Variable		
	DLM	Simulation	Relative error %
One-way clutch	.654091	.653422	+0.10238
One-way clutch*	.671709	.672153	-0.06605
Bike crank	.289424	.289697	-0.09423
Geometry block	.259250	.259183	+0.02585
Pawl and ratchet	.349660	.349436	+0.06410
Quick slider	.014484	.014485	-0.00690
Tape reel	.025586	.025590	-0.01563
Tape reel *	.032966	.032810	+0.47546
Positioner	.709387	.710589	-0.16916
3D crank slider	.035078	.035012	+0.18850

From Table 2, it can be seen that the assembly variations, as predicted by the DLM and Monte Carlo methods, are in close agreement. The DLM approximation appears to be very accurate in predicting assembly variations. For most problems, the error is less than a tenth of one percent.

Table 3 compares the predicted rejects for the linearized and simulated analyses. For most of the cases, the estimated assembly rejects on both limits by the DLM are good approximations to the simulated assembly rejects. However, the one-way clutch assembly shows a 2/1 ratio for the lower and upper limit rejects, while the linear estimation of the rejects is symmetric. This asymmetry may be caused by a highly nonlinear assembly constraint skewing the resulting distribution.

TABLE 3. SUMMARY OF REJECT RESULTS - DLM VS. SIMULATION

Assembly Name	Upper/Lower Rejects			Total Rejects	
	DLM	Simulation	Differences	DLM/Sim	Difference
One-way clutch	296/296	424/201	128/95	592/625	33
One-way clutch*	368/368	514/274	146/94	736/788	52
Bike crank	185/185	201/196	16/11	370/397	27
Geometry block	191/191	181/208	10/17	382/389	7
Pawl and ratchet	503/503	444/504	59/1	1006/948	58
Quick slider	1917/1917	1956/2089	39/172	3834/4045	211
Tape reel	245/245	257/239	12/6	490/496	6
Tape reel *	1448/1448	1420/1355	28/93	2896/2775	121
Positioner	154/154	158/172	4/18	308/330	22
3D crank slider	515/515	483/535	32/20	1030/1018	12

Effect of Nonlinear Assembly Functions

Nonlinear kinematic assembly constraints can produce mean shifts and skewed distributions in the assembly or kinematic variables. Table 4 lists the distribution parameters σ, β_1, β_2 and mean shifts for the controlled assembly variables of all the tested cases (sample size 100,000). σ describes the spread,, β_1 the skewness, and β_2 the peakedness of the distribution. For a Normal distribution, β_1 is zero and β_2 is equal to 3.0.

TABLE 4. SUMMARY OF MEAN SHIFTS AND DISTRIBUTION PARAMETERS

Assembly Name	Controlled Assembly Variable		
	Estimated σ	Mean shift (in σ units)	Distribution(β_1/β_2)
One-way clutch	0.218030	-0.019750	-0.104/3.0239
One-way clutch*	0.223903	-0.021554	-0.087/3.0173
Bike crank	0.096475	-0.000415	-0.005/3.0288
Geometry block	0.086417	+0.001620	+0.002/3.0026
Pawl and ratchet	0.116553	+0.006006	+0.014/2.9876
Quick slider	0.004828	+0.011392	+0.030/3.0046
Tape reel	0.008529	-0.001665	-0.009/3.0094
Tape reel *	0.010989	-0.002921	-0.011/3.0047
Positioner	0.236462	-0.004287	-0.009/3.0460
3D crank slider	0.011693	+0.003848	+0.011/3.3258
Normal Distribution			0.00/3.0000

In Table 4, β_1 and β_2 do not deviate much from a Normal distribution. The one-way clutch, however, exhibited 10 times greater mean shift and skewness than any other case. The clutch results also showed the most asymmetry of all eight test problems, as indicated by the difference in rejects at the upper and lower limits in Table 3. Variance estimates, presented in Table 2, seem unaffected by nonlinearity.

A mean shift may cause nonsymmetric assembly rejects, as illustrated in Figure 3. Using the Standard Normal Tables for area under a Normal distribution, a 0.02σ shift in the mean would change the clutch upper and lower rejects from 296/296 to 315/219. The rest of the shift, to 424/201, is presumed to be due to skewness.

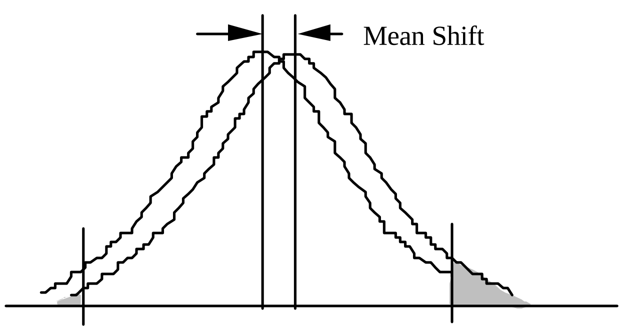

FIGURE 3. MEAN SHIFT AND ITS EFFECT ON ASSEMBLY REJECTS.

Effect of Sample Size on Assembly Variation

The accuracy of the simulated assembly variation depends upon the sample size. If the result of the assembly variation at sample size 100,000 is considered as the accurate solution, Table 5 lists the assembly variations at four different sample sizes for each of the examples, along with the relative error compared with the accurate solution. From the table, it can been seen that even for a small sample size, such as 1,000, the assembly variations are within 5% error, comparing to those at 100,000 sample sizes. Therefore, for assembly variations alone, small sample sizes, such as 1,000 to 5,000, can give acceptable results.

This effect of sample sizes on variation can be illustrated in Figure 4. The variation limits for the DLM are shown for comparison. From the figure, it can be seen that in order for the Monte Carlo simulation to be more accurate than the DLM in predicting assembly variation, the average sample size for this group of assemblies must be greater than 30,000.

TABLE 5. ESTIMATES OF ASSEMBLY VARIATION VS. SAMPLE SIZE

Assembly Name	Variation and % Error of Controlled Assembly Variable			
	100	1,000	10,000	100,000
One-way clutch	.650935(0.38%)	.643516(1.52%)	.654599(0.18%)	.653422
One-way clutch*	.712223(5.96%)	.689708(2.61%)	.677427(0.78%)	.672153
Bike crank	.304315(5.05%)	.293267(1.24%)	.287590(0.72%)	.289697
Geometry block	.272015(4.95%)	.260079(0.35%)	.258631(0.21%)	.259183
Pawl and ratchet	.352449(0.86%)	.356603(2.05%)	.349877(0.13%)	.349436
Quick slider	.014862(2.60%)	.014480(0.03%)	.014383(0.70%)	.014485
Tape reel	.021282(16.8%)	.025054(2.09%)	.025507(0.32%)	.025590
Tape reel *	.031067(5.31%)	.033235(1.30%)	.032509(0.92%)	.032810
Positioner	.690083(2.89%)	.699265(1.59%)	.705424(0.72%)	.710589
3D crank slider	.030339(13.3%)	.035053(0.12%)	.035091(0.23%)	.035012

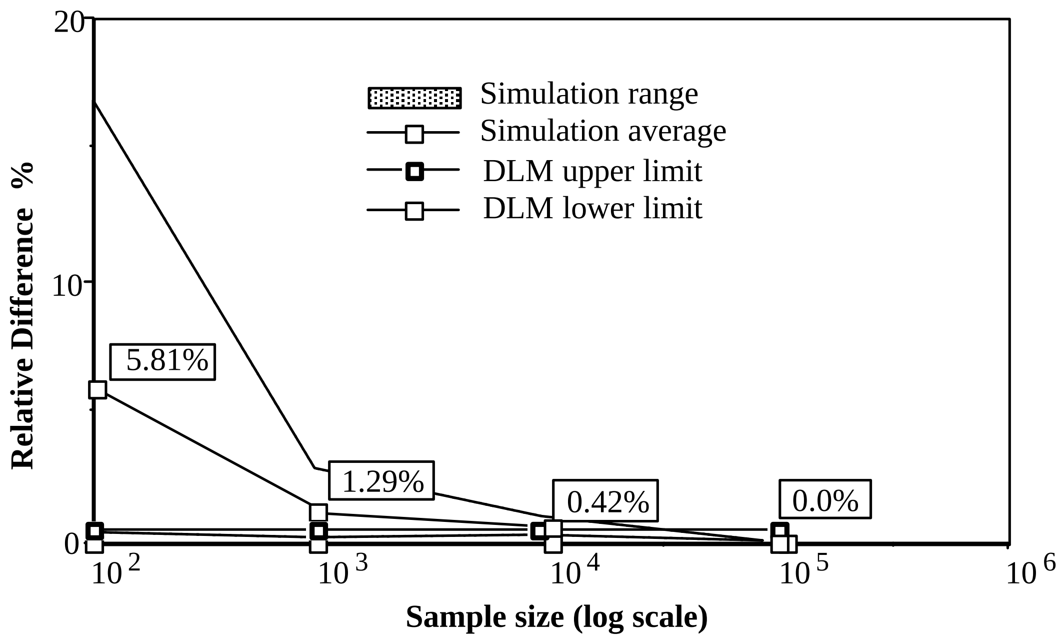

FIGURE 4. EFFECT OF SAMPLE SIZE ON PREDICTED VARIATION: SIMULATION VS. THE DLM

Effect of Sample Size on Assembly Rejects

The relatively small error between the assembly variation at the given sample size and the more accurate solution (at sample size 100,000) does not generally mean that the error of assembly rejects between these two cases is also small. Table 6 gives the assembly rejects for lower and upper limits, respectively, at different samples sizes. In comparing the relative error, the rejects should be proportionally extended to the number corresponding to a sample size of 100,000. For example, the rejects for the 10,000 sample size should be multiplied by 10 in order to compare with the 100,000 values.

From the tables, the conclusion can be made that even for a sample sizes of 10,000, the relative errors for many of the examples are still not acceptable. Judging from the accuracy on assembly rejects, large sample sizes, say 100,000 or even larger, are required to achieve reasonable precision.

Figure 5 shows the difference range plot of the assembly rejects for the cases tested by the Monte Carlo simulation and the DLM at various sample sizes. From the figure, it can be seen that the difference range shrinks very fast as the sample size increases. In order for the Monte Carlo simulation to be more accurate than the DLM in predicting assembly rejects for this group of assemblies, the sample size must be larger than 10,000. Otherwise, the Monte Carlo simulation may give poorer predictions about assembly rejects.

For accuracy at high quality levels, the sample size must be increased considerably, as shown in Table 7 (Shapiro and Gross, 1981).

TABLE 6. ESTIMATES OF ASSEMBLY REJECTS VS. SAMPLE SIZE (UPPER/LOWER LIMIT)

Assembly Name	Rejects at Upper/Lower Limits of Controlled Assembly Variable			
	100	1,000	10,000	100,000
One-way clutch	0/1	1/2	21/41	201/424
One-way clutch*	0/0	3/5	32/52	274/514
Bike crank	0/0	3/0	19/21	196/201
Geometry block	0/0	2/1	17/13	208/181
Pawl and ratchet	2/0	5/5	55/35	504/444
Quick slider	1/4	22/18	211/179	2089/1956
Tape reel	0/0	3/1	27/22	239/257
Tape reel *	1/2	20/19	133/136	1355/1420
Positioner	0/0	1/0	13/16	172/158
3D crank slider	0/0	6/4	54/42	535/483

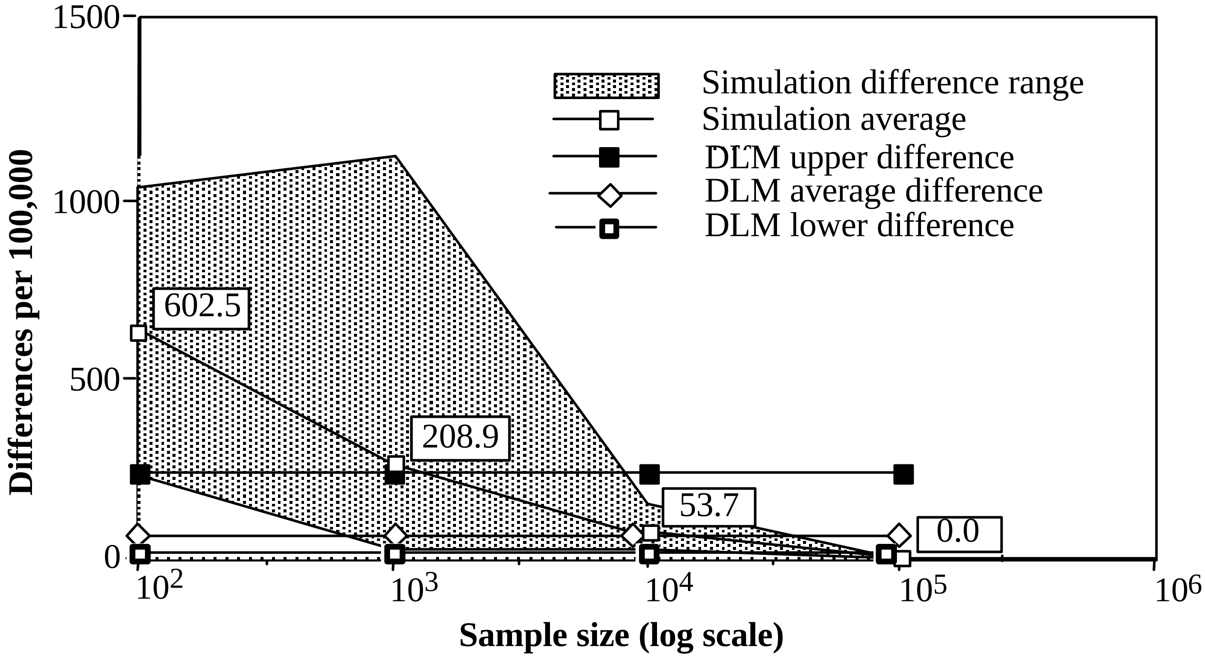

FIGURE 5. EFFECT OF SAMPLE SIZE ON TOTAL REJECTS: SIMULATION VS. THE DLM

TABLE 7. SAMPLE SIZE REQUIRED FOR MONTE CARLO SIMULATIONS

Assembly Yield	Rejects Fraction	Error in Rejects (±)		
		5%	10%	25%
0.9900	0.0100	107,000	27,000	4,300
0.9950	0.0050	215,000	54,000	8,600
0.9973	0.0027	400,000	100,000	16,000
0.9990	0.0010	1,081,000	270,000	43,000
0.9999	0.0001	10,823,000	2,706,000	433,000

Note: The confidence interval is 90%.

CONCLUSIONS

Seven 2-D and one 3-D mechanical assemblies, two of them with geometrical feature control tolerances, have been tested using Monte Carlo simulation and the Direct Linearization Method. The following conclusions can be obtained from the comparison of these methods:

1. The DLM is accurate in estimating the assembly variations. It is also accurate in most of the cases in predicting assembly rejects, except for highly nonlinear assembly constraints.

2. The sample size for Monte Carlo simulations has great influence in predicting assembly rejects, but has little effect on the simulated assembly variations once the sample size is larger than 1,000.

3. Nonlinear assembly constraints may cause a mean shift in the resultant assembly or kinematic dimensions and skewness in the distribution.

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