



## **Tolerance Allocation Methods for Designers**

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### **ABSTRACT**

Tolerance allocation is a design tool. It provides a rational basis for assigning tolerances to dimensions. Several algorithms are described in this paper for performing tolerance allocation, which is defined as the re-distribution of the “tolerance budget” within an assembly to reduce over-all cost of production, while meeting target levels for quality.

The task of placing +/- tolerances on each dimension of a CAD model or set of engineering drawings may seem menial and of little consequence. However, it can have enormous impact on cost and quality. On the engineering design side, it affects the fit and function of the final product, which can cause poor performance and dissatisfied customers. On the manufacturing side, tolerance requirements determine the selection of machines, tooling and fixtures; operator skill levels and setup costs; inspection precision and gaging; and scrap and rework. In short, it affects nearly every aspect of the product life cycle.

Using the new CAD-based tolerance analysis tools, designers can perform variation analysis on the CAD model, before parts are made or tooling purchased. They can determine where tolerance controls are needed and how tight the limits must be set, to assist in process planning and tool design. Using the same CAD model, production personnel can use tolerance tools to determine whether marginal or out-of-spec parts can still be used. Tolerance analysis tools provide a quantitative basis for design for manufacture decisions, resulting in shorter product development time and increased quality. There is probably no other design improvement effort which can yield greater benefits for less cost than the careful analysis and assignment of tolerances.

# Tolerance Allocation Methods for Designers

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## 1.0 Improving Quality by Tolerancing Techniques

There are several advanced tolerancing techniques available to a designer to improve quality levels in assemblies. They are primarily fine-tuning operations which can be applied to an assembly distribution to reduce the number of rejected assemblies, sometimes dramatically. The three methods to be presented here are:

- 1 Centering the Mean of the Assembly Distribution
- 2 Upper or Lower Limit Justification of the Mean
- 3 Reducing the Spread of the Distribution

When bringing a manufacturing process for a new product on line, small lots are produced to qualify the tooling. Parts are assembled, critical assembly features are measured and the mean and standard deviation of the resulting distributions are calculated. It is often found that the mean of the assembly distribution is not symmetrically distributed with respect to the upper and lower design specification limits (USL and LSL), resulting in large numbers of rejects at one or the other design limit. This can be caused by tooling bias, setup error, shrinkage of molded or cast parts, or other factors. If the errors are systematic, adjustments must be made to the tooling or processes to center the distribution. By centering the distribution, the total number of rejects can be minimized.

If only one limit is critical, as with a minimum clearance specification, the mean may be shifted away from the constraining limit until the desired quality level is reached. These operations are depicted in Fig. 1.

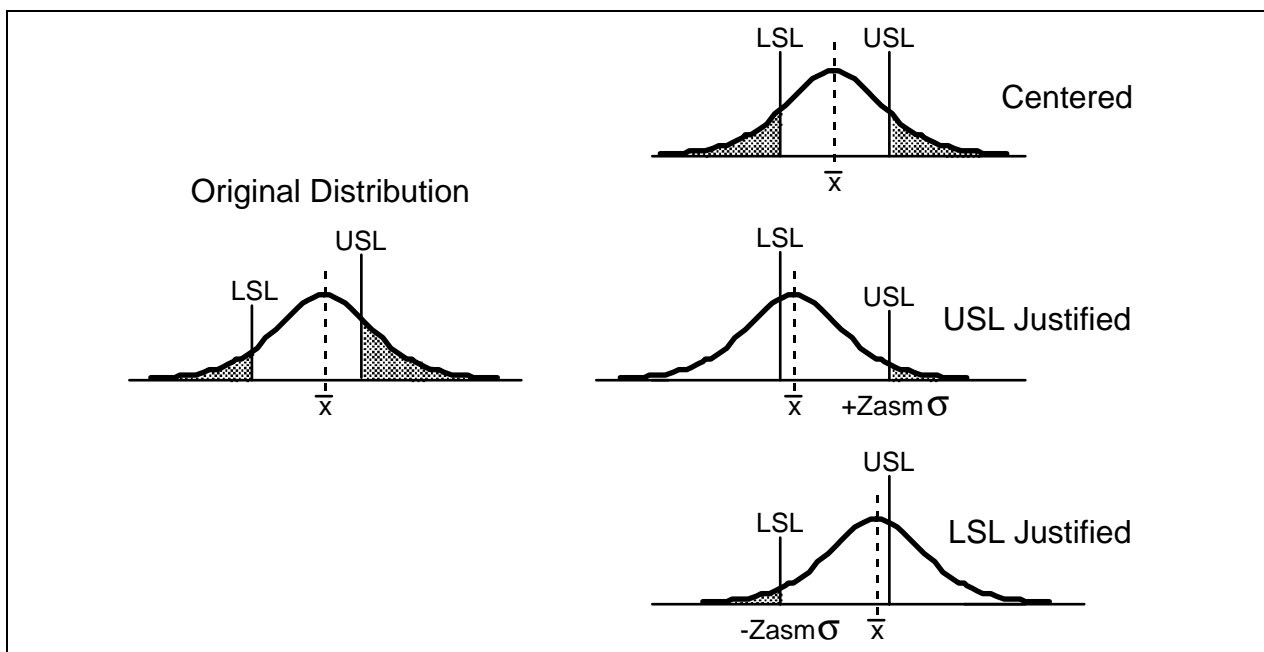


Figure 1. Effects of mean shift operations

## 1.1 Centering the Mean of the Assembly Distribution

Centering the mean puts the greatest distance between the tolerance limits and the mean, thus minimizing the number of rejects for a given assembly tolerance distribution. By adjusting one or more component nominals, the assembly mean may be moved to the center, halfway between the USL and LSL. Clearly, the centered distribution will have fewer rejects.

Note that adjustments to the mean do not involve tightening any tolerances, nor do tightening tolerances affect the mean. To tighten a tolerance may require switching to a more expensive process or creation of special fixtures, in which case adjusting the nominal dimension of a component may be less costly.

Deciding which component nominals to change is a design decision. Some components may be vendor-supplied and not allowed to change. Others may be too costly to change due to tooling modifications or other requirements. Some may simply require a change in the program of an NC machine used in its production. Of course, if the assembly is in production, the fewer parts you have to change, the better.

If more than one nominal is to be changed, a set of weight factors may be used for allocation of component nominals. The designer sets the value of the weight factors corresponding to those components he wishes to change. The relative weights determine how much change is assigned to each component.

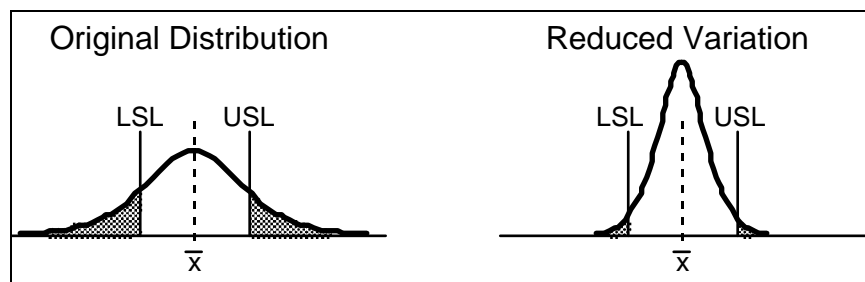
## 1.2 Upper or Lower Limit Justification of the Mean

Small changes may also be made to individual nominal dimensions to shift the nominal spec towards either the upper or lower spec limit. This may be desirable if there is only one spec limit or if one of the two spec limits is more important than the other. As with the centering allocation, the distribution is simply shifted without changing its spread. For example, a minimum clearance (LSL) may be critical to an assembly, but the maximum clearance may not be important.

In USL justification, the center of the distribution is shifted such that the  $+Z_{asm}\sigma$  limit matches with the USL, where  $\sigma$  is the standard deviation of the assembly variation and  $Z_{asm}$  is the desired quality level expressed in standard deviations. Similarly, in LSL justification, the distribution is shifted such that the  $-Z_{asm}\sigma$  limit matches with the LSL.

## 1.3 Variance Adjustment

If the mean has been centered and the reject rate is still too high because the spread of the distribution is too broad, it will be necessary to tighten tolerances on one or more assembly component dimensions to reduce the standard deviation of the assembly, as shown in Fig. 2.



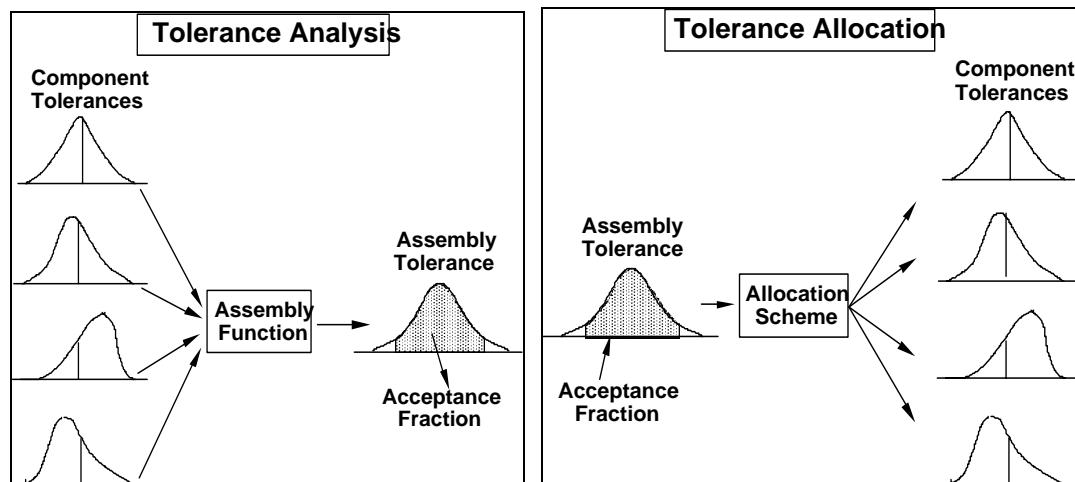
**Figure 2.** Effects of reduced variation

All three nominal allocation methods may be used in conjunction with any tolerance allocation method to simultaneously shift the mean and change the spread of the distribution.

## 2.0 Tolerance Analysis vs. Tolerance Allocation

The analytical modeling of assemblies provides a quantitative basis for the evaluation of design variations and specification of tolerances. An important distinction in tolerance specification is that engineers are more commonly faced with the problem of tolerance allocation rather than tolerance analysis.

The difference between these two problems is illustrated in Fig. 3. In tolerance analysis the component tolerances are all known or specified and the resulting assembly variation is calculated. In tolerance allocation, on the other hand, the assembly tolerance is known from design requirements, whereas the magnitude of the component tolerances to meet these requirements are unknown. The available assembly tolerance must be distributed or allocated among the components in some rational way. The influence of the tolerance accumulation model and the allocation rule chosen by the designer on the resulting tolerance allocation will be demonstrated.



**Figure 3.** Tolerance Analysis vs. Tolerance Allocation.

Another difference in the two problems is the yield or acceptance fraction of the assembly process. The assembly yield is the quality level. It is the percent of assemblies which meet the engineering tolerance requirements. It may be expressed as the percent of acceptable assemblies or the percent rejects. For high quality levels, the rejects may be expressed in parts-per-million (ppm), that is, the number of rejects per million assemblies .

In tolerance analysis the assembly yield is unknown. It is calculated by summing the component tolerances to determine the assembly variation, then applying the upper and lower spec limits to the calculated assembly distribution. In tolerance allocation, on the other hand, the assembly yield is specified as a design requirement. The component tolerances must then be set to assure that the resulting assembly yield meets the spec.

The rational allocation of component tolerances requires the establishment of a rule for distributing the assembly tolerance among the components. The following sections present several examples of useful rules.

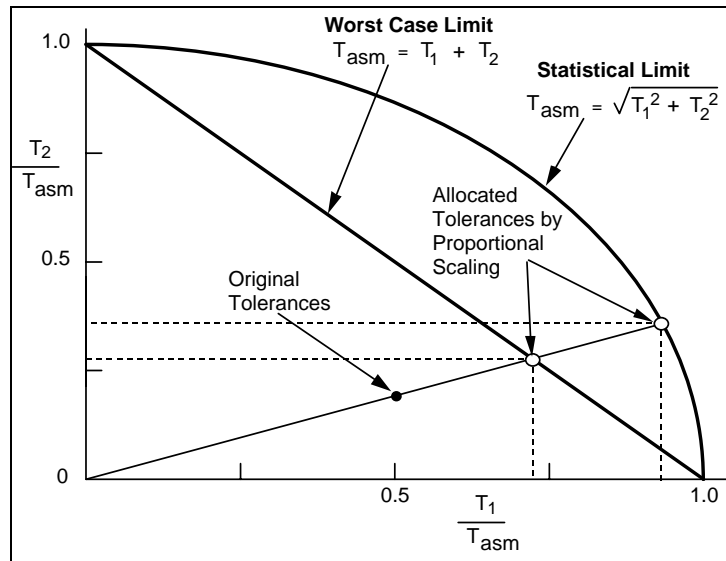
## 2.1 Allocation By Proportional Scaling

The designer begins by assigning reasonable component tolerances based on process or design guidelines. The component tolerances are summed to see if they meet the specified assembly tolerance. If not, the component tolerances are scaled by a constant proportionality factor. In this way the relative magnitudes of the component tolerances are preserved.

This algorithm is demonstrated graphically in Fig. 4. for an assembly tolerance  $T_{asm}$ , which is the sum of two component tolerances,  $T_1$  and  $T_2$ . The straight line labeled as the **Worst Case Limit** is the locus of all possible combinations of  $T_1$  and  $T_2$  which, added linearly, equal  $T_{asm}$ . The ellipse labeled **Statistical Limit** is the locus of root sum squares of  $T_1$  and  $T_2$  which equal  $T_{asm}$ .

Suppose the designer chooses initial values for  $T_1$  and  $T_2$  based on typical process tolerances for the two component parts. This combination is the point labeled **Original Tolerances** in the figure. By drawing a line from the origin through this point, then extending it until it intersects the Worst Case Limit, the largest possible tolerances for  $T_1$  and  $T_2$  are obtained, which satisfy the worst case condition and which still have the original ratio of  $T_1$  to  $T_2$ . Extending this line further, until it intersects the Statistical Limit curve, new values for  $T_1$  and  $T_2$  are obtained which satisfy the assembly tolerance limit by root sum squares.

Although Fig. 4 illustrates 1-D tolerance accumulation models, the algorithm may also be applied to 2-D or 3-D tolerance accumulation by pre-multiplying each component tolerance by its tolerance sensitivity.



**Figure 4.** Graphical interpretation of tolerance allocation by proportional scaling.

## 2.2 1-D Example: Worst Case Tolerance Allocation by Proportional Scaling

The following example is based on the shaft and housing assembly shown in Fig. 5. Two bearing sleeves maintain the spacing of the bearings to match that of the shaft. Accumulation of variation in the assembly results in variation in the end clearance. Positive clearance is required.

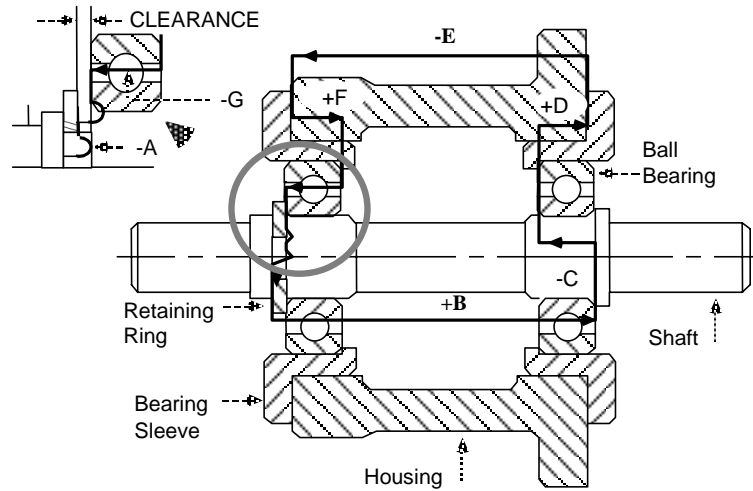


Figure 5 Shaft and housing assembly.

Initial tolerances for parts **B**, **D**, **E**, and **F** are selected from tolerance guidelines such as those illustrated in Fig. 6. The bar chart shows the typical range of tolerance for several common processes. The numerical values appear in the table above the bar chart. Each row of the numerical table corresponds to different nominal size. For example, a turned part having a nominal dimension of 0.750 in. can be produced to a tolerance ranging from  $\pm 0.001$  to  $\pm 0.006$  in., depending on the number of passes, rigidity of the machine and fixtures, etc. Tolerances are chosen initially from the middle of the range for each dimension and process, then adjusted to match the design limits and reduce production costs.

RANGE OF SIZES		TOLERANCES $\pm$								
FROM	THROUGH									
0.000	0.599	0.00015	0.0002	0.0003	0.0005	0.0008	0.0012	0.002	0.003	0.005
0.600	0.999	0.00015	0.00025	0.0004	0.0006	0.001	0.0015	0.0025	0.004	0.006
1.000	1.499	0.0002	0.0003	0.0005	0.0008	0.0012	0.002	0.003	0.005	0.008
1.500	2.799	0.00025	0.0004	0.0006	0.001	0.0015	0.0025	0.004	0.006	0.010
2.800	4.499	0.0003	0.0005	0.0008	0.0012	0.002	0.003	0.005	0.008	0.012
4.500	7.799	0.0004	0.0006	0.001	0.0015	0.0025	0.004	0.006	0.010	0.015
7.800	13.599	0.0005	0.0008	0.0012	0.002	0.003	0.005	0.008	0.012	0.020
13.600	20.999	0.0006	0.001	0.0015	0.0025	0.004	0.006	0.010	0.015	0.025
LAPPING & HONING										
DIAMOND TURNING & GRINDING										
BROACHING										
REAMING										
TURNING, BORING, SLOTTING, PLANING, & SHAPING										
MILLING										
DRILLING										

Figure 6. Tolerance range of machining processes [Trucks, 1987].

The box below shows the problem setup. The retaining ring (**A**) and the two bearings (**C** and **G**) supporting the shaft are vendor-supplied, hence their tolerances are fixed and must not be altered by the allocation process. Initial values for **B**, **D**, **E** and **F** are shown as selected from the table, assuming a mid-range tolerance for turned parts. The critical clearance is the shaft end-play, which is determined by tolerance accumulation in the assembly. The vector diagram overlaid on the figure is the assembly loop that models the end-play. The average clearance is the vector sum of the average part dimensions in the loop:

<b>Table 1 Initial Tolerance Specifications</b>							
Required Clearance	= .020±.015						
Average Clearance	= -A + B - C + D - E + F - G						
	= -.0505 + 8.000 - .5093 + .400 - 7.711 + .400 - .5093						
	= .020						
<b>Dimension</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>
Nominals	.0505	8.000	.5093	.400	7.711	.400	.5093
Tolerances(±)							
Design		.008		.002	.006	.002	
Fixed	.0015		.0025				.0025

The clearance tolerance is obtained by summing the component tolerances by worst case:

$$\begin{aligned}
 T_{SUM} &= + T_A + T_B + T_C + T_D + T_E + T_F + T_G \\
 &= + .0015 + .008 + .0025 + .002 + .006 + .002 + .0025 \\
 &= .0245 \text{ (too large)}
 \end{aligned}$$

Now, solving for the proportionality factor:

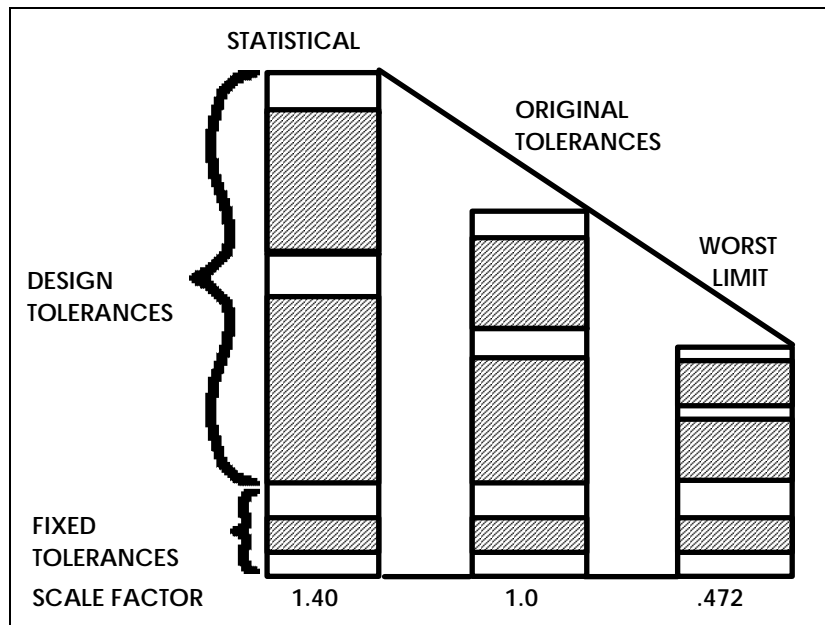
$$\begin{aligned}
 T_{ASM} &= .015 = .0015 + .0025 + .0025 + P (.008 + .002 + .006 + .002) \\
 P &= .47222
 \end{aligned}$$

Note that the three fixed tolerances were subtracted from the assembly tolerance before computing the scale factor. Thus, only the four design tolerances are re-allocated:

$$\begin{aligned}
 T_B &= .47222 (.008) = .00378 & T_E &= .47222 (.006) = .00283 \\
 T_D &= .47222 (.002) = .00094 & T_F &= .47222 (.002) = .00094
 \end{aligned}$$

Each of the design tolerances has been scaled down to meet assembly requirements as shown in Fig. 7. This procedure could also be followed assuming a statistical sum for the assembly tolerance, in which case the tolerances would be scaled up. Results are summarized in Table 2.



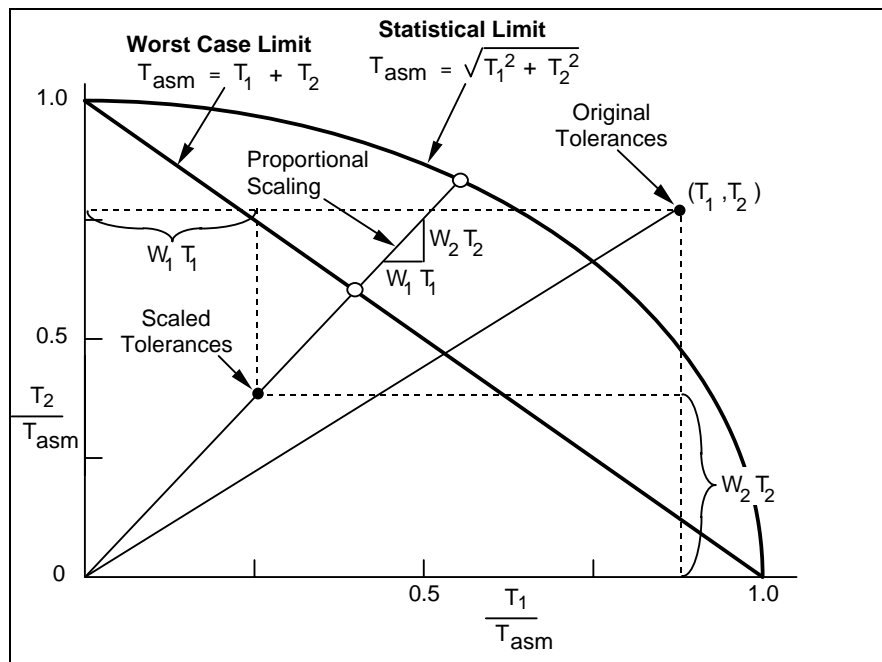


**Figure 7** Tolerance allocation by proportional scaling.

### 3.0 Allocation by Weight Factors

A more versatile method of assigning tolerances is by means of weight factors. Using this algorithm, the designer assigns weight factors to each tolerance in the chain and the system distributes a corresponding fraction of the tolerance pool to each component. A larger weight factor for a given component means a larger fraction of the tolerance pool will be allocated to it. In this way, more tolerance can be given to those dimensions which are the more costly or difficult to hold, thus improving the producibility of the design.

Fig. 8 illustrates this algorithm graphically for a two component assembly. The original values for component tolerances  $T_1$  and  $T_2$  are selected from process considerations and are represented as a point in the figure, as before. The tolerances are scaled, similar to proportional scaling, only the scale factor is weighted for each component tolerance so the greater scale factors yield the least reduction in tolerance.



**Figure 8.** Graphical interpretation of tolerance allocation by weight factors.

Again, although the figure illustrates 1-D tolerance accumulation models, the algorithm may be applied equally well to 2-D and 3-D stacks. It may also be applied to worst case, statistical, or six sigma tolerance sums.

Note that any components which are vendor-supplied, or subject to other design considerations, can be excluded from the allocation process by declaring them to be "fixed".

#### 4.0 1-D Example: Worst Case (WC) Allocation by Weight Factors

The shaft and housing assembly of Section 2.0 will be revisited, using weight factors to allocate tolerances. The three tolerances A, C and G are fixed. Design tolerances B, D, E and F are assigned weight factors of 10, 20, 10 and 20, respectively. Weights were determined on the basis of machining difficulty.

The clearance tolerance is obtained by summing the component tolerances by worst limits:

$$\begin{aligned}
 T_{\text{SUM}} &= + T_A + T_B + T_C + T_D + T_E + T_F + T_G \\
 &= + .0015 + .008 + .0025 + .002 + .006 + .002 + .0025 \\
 &= .0245 \text{ (too large)}
 \end{aligned}$$

Solving for the proportionality factor with weight factors:

$$\begin{aligned}
 T_i' &= P W_i T_i \\
 T_{\text{ASM}} &= .015 = .0015 + .0025 + .0025 + \\
 P &[(10/60)(.008) + (20/60)(.002) + (10/60)(.006) + (20/60)(.002)] \\
 P &= 2.31818
 \end{aligned}$$

Note that only the four design tolerances are re-allocated:

$$T_B = 2.31818(10/60)(.008) = .00309 \quad T_E = 2.31818(10/60)(.006) = .00232$$

$$T_D = 2.31818(20/60)(.002) = .00155 \quad T_F = 2.31818(20/60)(.002) = .00155$$

### 5.0 1-D Example: Statistical (RSS) Allocation by Weight Factors

The clearance tolerance is obtained by summing the component tolerances by root sum squares:

$$\begin{aligned} T_{SUM} &= \sqrt{T_A^2 + T_B^2 + T_C^2 + T_D^2 + T_E^2 + T_F^2 + T_G^2} \\ &= \sqrt{.0015^2 + .008^2 + .0025^2 + .002^2 + .006^2 + .002^2 + .0025^2} \\ &= .01108 \text{ (too small)} \end{aligned}$$

Solving for the proportionality factor with weight factors:

$$T_i' = PW_i T_i$$

$$T_{ASM} = \sqrt{T_A'^2 + P^2 W_B^2 T_B^2 + T_C^2 + P^2 W_D^2 T_D^2 + P^2 W_E^2 T_E^2 + P^2 W_F^2 T_F^2 + T_G^2}$$

$$P = 7.57238$$

Note that the three fixed tolerances are not re-allocated:

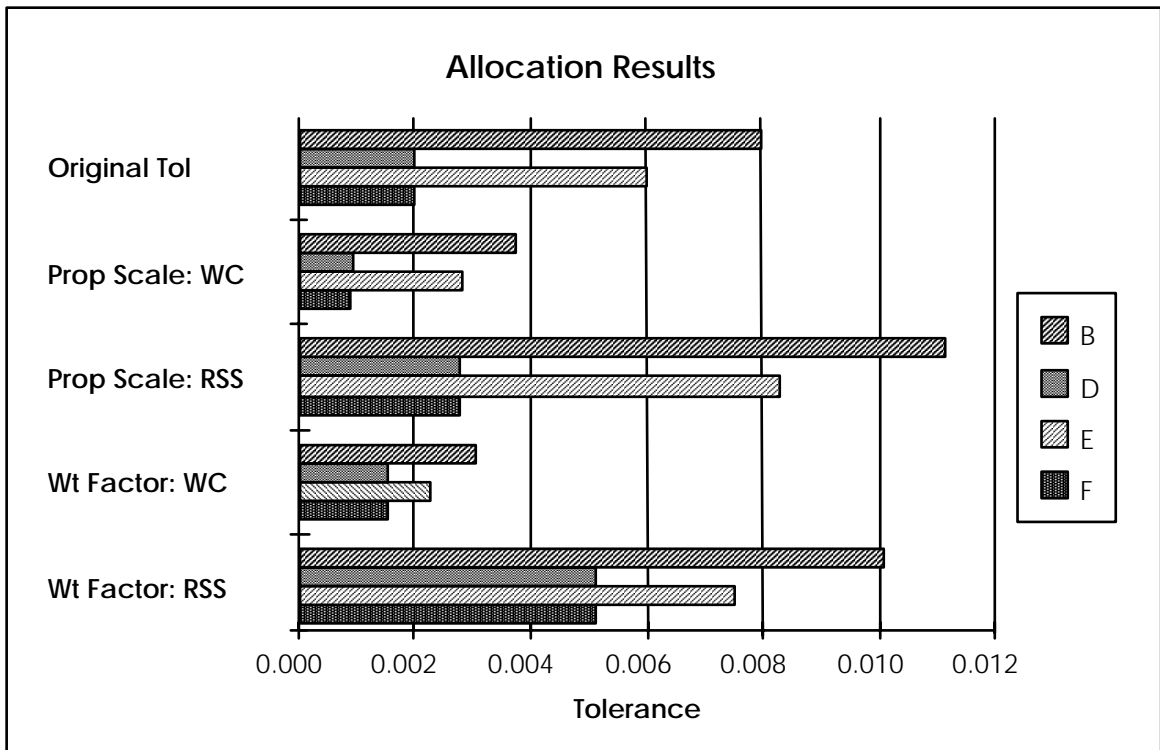
$$T_B = (7.57238)(10/60)(.008) = .01010 \quad T_E = (7.57238)(10/60)(.006) = .00757$$

$$T_D = (7.57238)(20/60)(.002) = .00505 \quad T_F = (7.57238)(20/60)(.002) = .00505$$

All of the preceding examples of allocation are compared in Table 2. A graphical comparison is shown in Fig. 9.

**Table 2 Comparison of Allocation Methods**

		Proportional		Weight Factors		
Dim	Original Tolerance	Worst Case	Stat $\pm 3\sigma$	Weight Factor	Worst Case	Stat $\pm 3\sigma$
A	.0015*	.0015	.0015	0	.0015	.0015
B	.008	.00378	.01116	10	.00309	.0101
C	.0025*	.0025	.0025	0	.0025	.0025
D	.002	.00094	.00279	20	.00155	.00505
E	.006	.00283	.00837	10	.00232	.00757
F	.002	.00094	.00279	20	.00155	.00505
G	.0025*	.0025	.0025	0	.0025	.0025
<b>Assembly Tolerance</b>		.0150	.0150		.0150	.0150
<b>Scale Factor (P)</b>		.47222	1.39526		2.31818	7.57238
*Fixed tolerances						

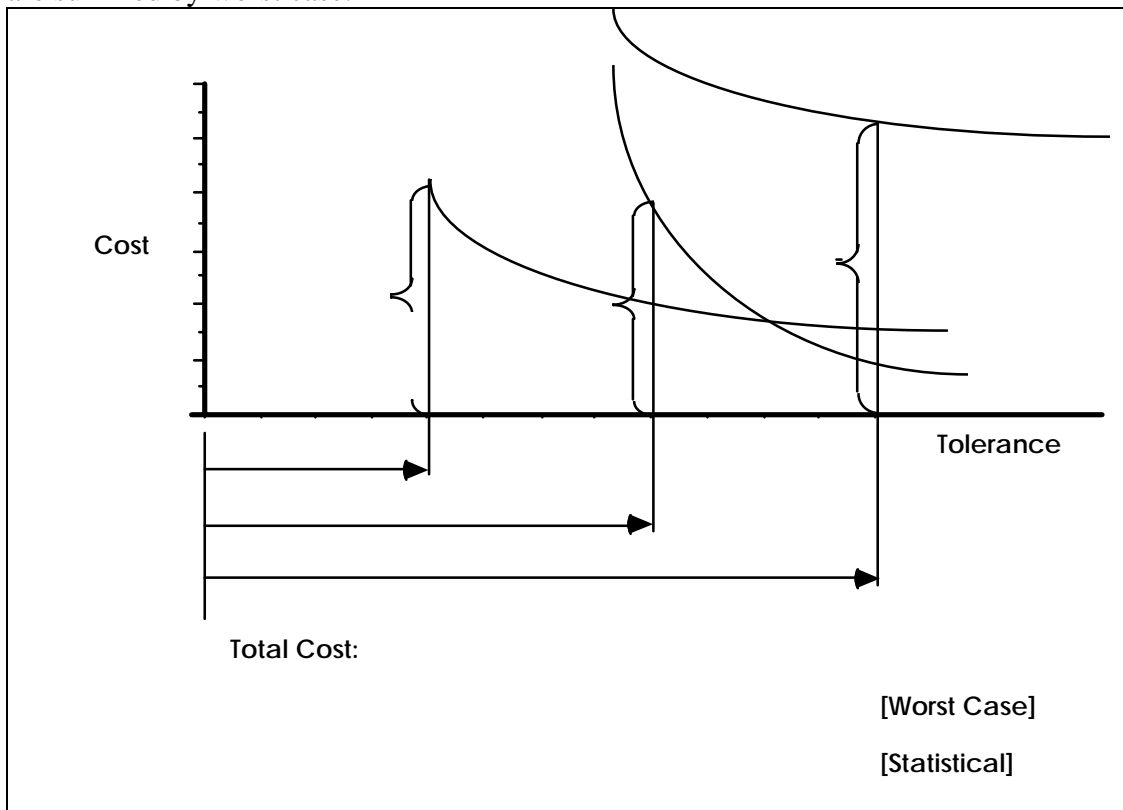


**Figure 9.** Tolerance allocation by proportional scaling and weight factors

## 6.0 Tolerance Allocation Using Least Cost Optimization

A promising method of tolerance allocation uses optimization techniques to assign component tolerances such that the cost of production of an assembly is minimized. This is accomplished by defining a cost-vs.-tolerance curve for each component part in the assembly. The optimization algorithm varies the tolerance for each component and searches systematically for the combination of tolerances which minimizes the cost.

Fig. 10 illustrates the concept simply for a three component assembly. Three cost-vs.-tolerance curves are shown. Three tolerances ( $T_1, T_2, T_3$ ) are initially selected. The corresponding cost of production is  $C_1 + C_2 + C_3$ . The optimization algorithm tries to increase the tolerances to reduce cost; however, the specified assembly tolerance limits the tolerance size. If tolerance  $T_1$  is increased, then tolerance  $T_2$  or  $T_3$  must decrease to keep from violating the assembly tolerance constraint. It is difficult to tell by inspection which combination will be optimum, but you can see from the figure that a decrease in  $T_2$  results in a significant decrease in cost, while a corresponding decrease in  $T_3$  results in smaller increase in cost. In this manner, one could manually adjust tolerances until no further cost reduction is achieved. The optimization algorithm is designed to find it with a minimum of iteration. Note that the values of the set of optimum tolerances will be different when the tolerances are summed statistically than when they are summed by worst case.



**Figure 10.** Optimal tolerance allocation for minimum cost.

A necessary factor in optimum tolerance allocation is the specification of cost-vs.-tolerance functions. Several algebraic functions have been proposed, as summarized in Table 3. The Reciprocal Power function:  $C = A + B/\text{tol}^k$  includes the Reciprocal and Reciprocal Squared rules for integer powers of  $k$ . The constant coefficient  $A$  represents fixed costs. It may include setup

cost, tooling, material, prior operations, etc. The **B** term determines the cost of producing a single component dimension to a specified tolerance and includes the charge rate of the machine. Costs are calculated on a per part basis. When tighter tolerances are called for, speeds and feeds may be reduced and the number of passes increased, requiring more time and higher costs. The exponent **k** describes how sensitive the process cost is to changes in tolerance specifications.

**Table 3. Proposed Cost-of-Tolerance Models**

Cost Model	Function	Author	Ref
Reciprocal Squared	$A + B/\text{tol}^2$	Spotts	[Spotts 1973]
Reciprocal	$A + B/\text{tol}$	Chase&Greenwood	[Chase 1988]
Reciprocal Power	$A + B/\text{tol}^k$	Chase et. al.	[Chase 1989]
Exponential	$A e^{-B(\text{tol})}$	Speckhart	[Speckhart 1972]

Little has been done to verify the form of these curves. Manufacturing cost data are not published since they are so site-dependent. Even companies using the same machines would have different costs for labor, materials, tooling and overhead.

A study of cost vs. tolerance was made for the metal removal processes over the full range of nominal dimensions. This data has been curve fit to obtain empirical functions. The form was found to follow the reciprocal power law. The results are presented in the appendix to this chapter. The original cost study is decades old and may not apply to modern N/C machines.

A closed-form solution for the least-cost component tolerances was developed by Spotts [1973]. He used the method of Lagrange Multipliers, assuming a cost function of the form  $C=A+B/\text{tol}^2$ . Chase extended this to cost functions of the form  $C=A+B/\text{tol}^k$  as follows [Chase et al. 1989]:

$$\begin{aligned} \frac{\dot{C}}{\dot{T}_i} (\text{Cost function}) + \lambda \frac{\dot{C}}{\dot{T}_i} (\text{Constraint}) &= 0 \quad (i = 1, \dots, n) \\ \frac{\dot{C}}{\dot{T}_i} (\bullet (A_j + B_j/T_j^{k_j})) + \lambda \frac{\dot{C}}{\dot{T}_i} (\bullet T_j^2 - T_{asm}^2) &= 0 \quad (i = 1, \dots, n) \\ \lambda &= \frac{k_i B_i}{2 T_i^{(k_i+2)}} \quad (i = 1, \dots, n) \end{aligned}$$

Eliminating  $\lambda$  by expressing it in terms of  $T_1$  (arbitrarily selected):

$$T_i = \left( \frac{k_i B_i}{k_1 B_1} \right)^{1/(k_i+2)} \cdot T_1^{(k_1+2)/(k_i+2)} \quad (1)$$

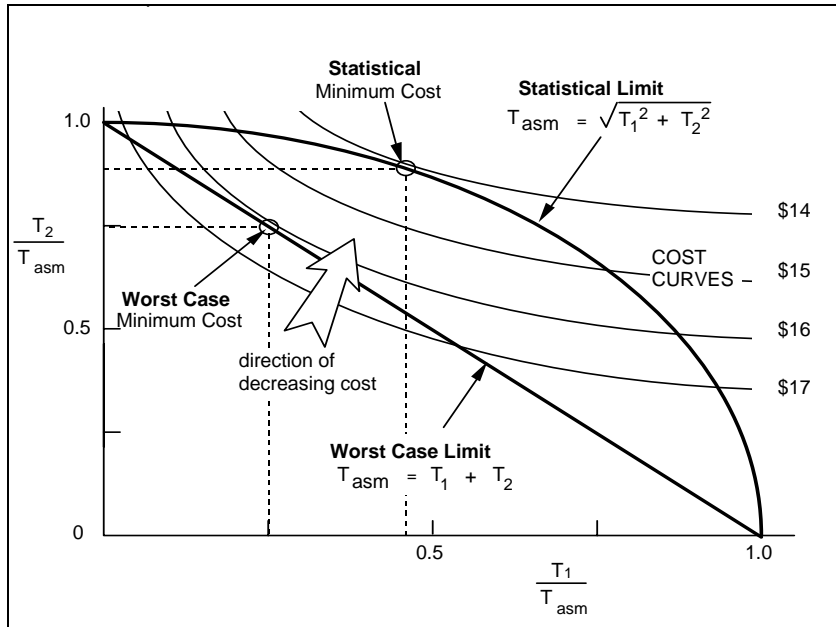
Substituting for each of the  $T_i$  in the assembly tolerance sum:

$$T_{ASM}^2 = T_1^2 + \bullet \left( \frac{k_i B_i}{k_1 B_1} \right)^{2/(k_i+2)} \cdot T_1^{2(k_1+2)/(k_i+2)} \quad (2)$$

The only unknown in Eq (2) is  $T_1$ . One only needs to iterate the value of  $T_1$  until both sides of Eq (2) are equal to obtain the minimum cost tolerances. A similar derivation based on a worst case assembly tolerance sum yields:

$$T_{ASM} = T_1 + \bullet \left( \frac{k_i B_i}{k_1 B_1} \right)^{1/(k_i+1)} \cdot T_1^{(k_1+1)/(k_i+1)} \quad (3)$$

A graphical interpretation of this method is shown in Fig. 11. Note that as the method of Lagrange Multipliers assumes, the constant cost curve is tangent to the tolerance limit curve for the minimum cost tolerance value.



**Figure 11.** Graphical interpretation of minimum cost tolerance allocation.

Numerical results for the 1-D shaft and housing example problem are shown in Table 4:

**Table 4. Minimum Cost Tolerance Allocation**

Dimension	Tolerance Cost Data			Original Tolerance	Allocated Tolerances	
	Setup A	Coefficient B	Exponent k		Worst Case	Stat. $\pm 3\sigma$
A		*	*	.0015*	.0015*	.0015*
B	\$1.00	0.15997	-0.43899	.008	.00254	.0081
C		*	*	.0025*	.0025*	.0025*
D	1.00	0.07202	-0.46823	.002	.001736	.00637
E	1.00	0.12576	-0.46537	.006	.002498	.00792
F	1.00	0.07202	-0.46823	.002	.001736	.00637
G		*	*	.0025*	.0025*	.0025*
Assembly Variation				.0245(WC) .0111(RSS)	.0150(WC)	.0150(RSS)
Assembly Cost				\$9.34	\$11.07	\$8.06
Acceptance Fraction					1.000	.9973
"True Cost"					\$11.07	\$8.08

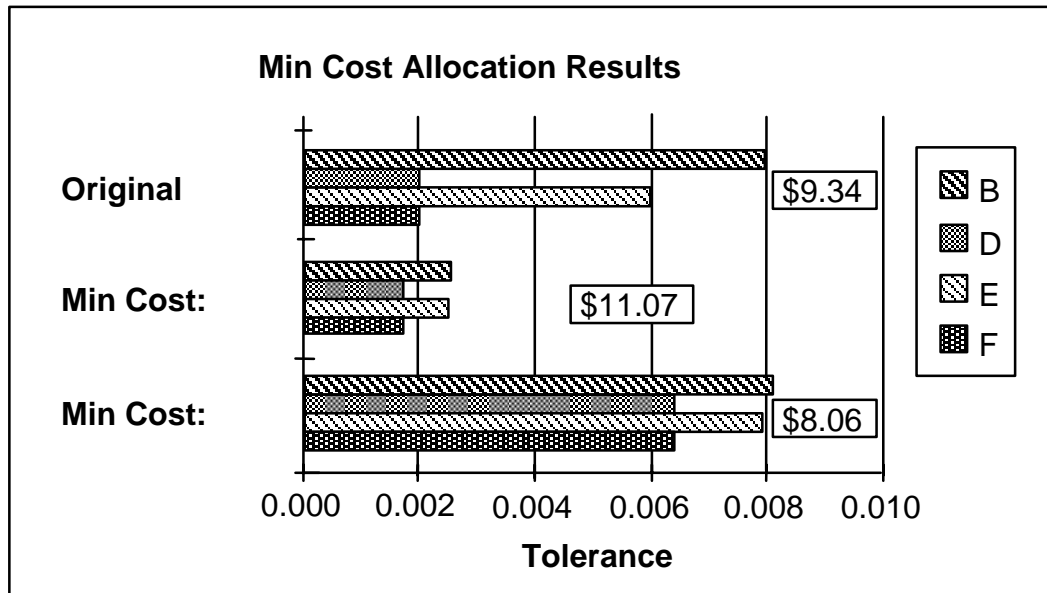
\*Fixed tolerances

The Setup Cost is coefficient A in the cost function. Setup cost does not affect the optimization. For this example, the setup costs were all chosen as equal, so they would not mask the affect of the tolerance allocation. In this case, they merely added \$4.00 to the assembly cost for each case.

Parts A, C and G are vendor-supplied. Since their tolerances are fixed, their cost cannot be changed by re-allocation, so no cost data is included in the table.

Note that in this example the assembly cost increased when worst case allocation was performed. This is due to the fact that the original tolerances, when summed by worst case, gives an assembly variation of 0.0245 in. This exceeded the specified assembly tolerance limit of 0.015 in. Thus, the component tolerances had to be tightened, driving the cost up. When summed

statistically, however, the assembly variation was only .00111 in., which was less than the spec limit, allowing the allocation algorithm to increase the component tolerances, driving the cost down. A graphical comparison is shown in Fig. 12. It is clear from the graph that tolerances for B and E were reduced the most in the Worst Case model, while D and F were increased more in the Statistical model.



**Figure 12.** Comparison of minimum cost allocation results

The advantages of the Lagrange multiplier method are:

- 1) It eliminates the need for multiple-parameter iterative solutions
- 2) It can handle either worst case or statistical assembly models
- 3) It allows alternative cost-tolerance models

The limitations are:

- 1) Tolerance limits cannot be imposed on the processes. Most processes are only capable of a specified range of tolerance. The designer must check the resulting component tolerances to make sure they are within the range of the process.
- 2) It cannot readily treat the problem of simultaneously optimizing an assembly with interdependent design specifications. That is, when an assembly has more than one design specification, with common component dimensions contributing to each spec, some iteration is required to find a set of shared tolerances which satisfies each of the engineering requirements.

Problems which exhibit these characteristics may be optimized using nonlinear programming techniques or by optimizing tolerances for one assembly spec at a time, then choosing the lowest set of component tolerance values required to satisfy all assembly specs simultaneously.

## 7.0 True Cost and Optimum Acceptance Fraction

The "True Cost" in Table 5 is defined as the total cost of an assembly divided by the acceptance fraction or yield. Thus, the "true cost" is adjusted to include a share of the cost of the rejected assemblies. It does not include, however, any parts which might be saved by re-work or the cost of rejecting individual component parts.



An interesting exercise is to calculate the optimum acceptance fraction, that is, the rejection rate which would result in the minimum True Cost. This requires an iterative solution. For the example problem, the results are shown in Table 5:

**Table 5. Minimum True Cost**

<b>Cost Model</b>	<b><math>\Sigma A</math></b>	<b>Zasm</b>	<b>Opt. Accept Fract</b>	<b>True Cost</b>
A + B/tol <sup>k</sup>	\$4.00	2.03	.9576	\$7.67
A + B/tol <sup>k</sup>	\$8.00	2.25	.9756	\$11.82

The interpretation of these results is that loosening up the tolerances will save money on production costs, but will increase the cost of rejects. By iterating on the acceptance fraction, it is possible to find the value which minimizes the combined cost of production and rejects. Note, however, that the setup costs were set very low. If setup costs were doubled, as shown in the second row of the table, the cost of rejects would be higher, requiring a higher acceptance level.

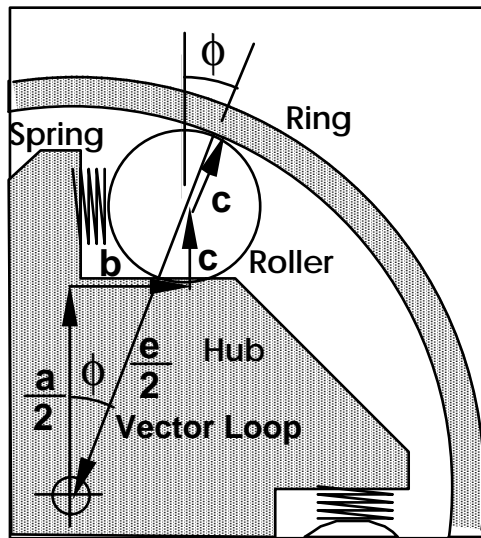
In the very probable case where individual process cost vs. tolerance curves are not available, an optimum acceptance fraction for the assembly could be based instead on more-available cost per reject data. This optimum acceptance fraction could then be used in conjunction with allocation by proportional scaling or weight factors to provide a meaningful cost-related alternative to allocation by least cost optimization.

## **8.0 2-D and 3-D Tolerance Allocation**

Tolerance allocation may be applied to 2-D and 3-D assemblies as readily as 1-D. The only difference is that each component tolerance must be multiplied by its tolerance sensitivity, derived from the geometry by small variations. The proportionality factors, weight factors, cost factors are still obtained as described above, with sensitivities inserted appropriately.

### **8.1 One-way Clutch Assembly**

The application of tolerance allocation to a 2-D assembly will be demonstrated on the one-way clutch assembly shown in Fig. 13. The clutch consists of four different parts: a hub, a ring, four rollers, and four springs. Only a quarter section is shown because of symmetry. During operation, the springs push the rollers into the wedge-shaped space between the ring and the hub. If the hub is turned counter-clockwise, the rollers bind, causing the ring to turn with the hub. When the hub is turned clockwise, the rollers slip, so torque is not transmitted to the ring. A common application for the clutch is a lawn mower starter [Fortini 1967].



**Figure 13** Clutch Assembly with Vector Loop

The contact angle  $\phi$ , between the roller and the ring, is critical to the performance of the clutch. The angle  $\phi$  and variable  $b$ , the location of contact between the roller and the hub, are both dependent assembly variables. The magnitude of  $\phi$  and  $b$  will vary from one assembly to the next due to the variations of the component dimensions  $a$ ,  $c$ , and  $e$ . Dimension  $a$  is the width of the hub;  $c$  and  $e/2$  are the radii of the roller and ring, respectively. A complex assembly function determines how much each dimension contributes to the variation of angle  $\phi$ . The nominal contact angle, when all of the independent variables are at their mean values, is 7.0 degrees. For proper performance, the angle must not vary more than  $\pm 1.0$  degree from nominal. These are the engineering design limits.

The objective of variation analysis for the clutch assembly is to determine the variation of the contact angle relative to the design limits. Table 6 below shows the nominal value and tolerance for the three independent dimensions which contribute to tolerance stackup in the assembly. Each of the independent variables is assumed to be statistically independent (not correlated with each other) and a normally distributed random variable. The tolerances are assumed to be  $\pm 3\sigma$ .

**Table 6: Independent Dimensions for the Clutch Assembly**

Dimension	Nominal	Tolerance
Hub width - $a$	2.1768 in	0.004 in
Roller radius - $c$	0.450 in	0.0004 in
Ring diameter - $e$	4.000 in	0.0008 in

### Vector Loop Model and Assembly Function for the Clutch

The vector loop method [Chase 1995], uses the assembly drawing as the starting point. Vectors are drawn from part-to-part in the assembly, passing through the points of contact. The vectors represent the independent and dependent dimensions which contribute to tolerance stackup in the assembly. Fig. 13, above, shows the resulting vector loop for a quarter section of the clutch assembly.

The vectors pass through the points of contact between the three parts in the assembly. Since the roller is tangent to the ring, both the roller radius  $c$  and the ring radius  $e/2$  are colinear. Once the vector loop is defined, the implicit equations for the assembly can easily be extracted. Eq. 4 shows the set of scalar equations for the clutch assembly derived from the vector loop.  $h_x$  and  $h_y$  are the sum of vector components in the  $x$  and  $y$  directions. A third equation,  $h_\theta$ , is the sum of relative angles between consecutive vectors, but it vanishes identically.

$$\begin{aligned} h_x = 0 &= b + c \sin(\phi) - e \sin(\phi) \\ h_y = 0 &= a + c + c \cos(\phi) - e \cos(\phi) \end{aligned} \quad (4)$$

Equations 16.4 may be solved for  $\phi$  explicitly:

$$\phi = \cos^{-1}\left(\frac{a + c}{e - c}\right) \quad (5)$$

The sensitivity matrix [S] can be calculated from equation 16.5 by differentiation or by finite difference:

$$[S] = \begin{bmatrix} \frac{\partial f}{\partial a} & \frac{\partial f}{\partial c} & \frac{\partial f}{\partial e} \\ \frac{\partial b}{\partial a} & \frac{\partial b}{\partial c} & \frac{\partial b}{\partial e} \end{bmatrix} = \begin{bmatrix} -2.6469 & -10.5483 & 2.6272 \\ -103.43 & -440.69 & 104.21 \end{bmatrix} \quad (6)$$

The top row of [S] are the tolerance sensitivities for  $\delta\phi$ . Assembly variations accumulate or stackup statistically by root-sum-squares:

$$\begin{aligned} \delta\phi &= \sqrt{\sum((S_{ij} \delta x_j)^2)} \\ &= \sqrt{(S_{11} \delta a)^2 + (S_{12} \delta c)^2 + (S_{13} \delta e)^2} \\ &= \sqrt{(-2.6469 \cdot 0.004)^2 + (-10.5483 \cdot 0.0004)^2 + (2.6272 \cdot 0.0008)^2} \\ &= 0.01159 \text{ radians} = 0.664 \text{ degrees} \end{aligned} \quad (7)$$

where  $\delta\phi$  is the predicted  $3\sigma$  variation,  $\delta x_j$  is the set of  $3\sigma$  component variations.

By worst case:

$$\begin{aligned}
 \delta\phi &= \sum |S_{ij}| \delta x_j & (8) \\
 &= |S_{11}| \delta a + |S_{12}| \delta c + |S_{13}| \delta e \\
 &= 2.6469 \cdot 0.004 + 10.5483 \cdot 0.0004 + 2.6272 \cdot 0.0008 \\
 &= 0.01691 \text{ radians} = 0.9688 \text{ degrees}
 \end{aligned}$$

where  $\delta\phi$  is the predicted extreme variation.

## 8.2 Allocation by Scaling, Weight Factors

Once you have RSS and worst case expressions for the predicted variation  $\delta\phi$ , you may begin applying various allocation algorithms to search for a better set of design tolerances. As we try various combinations, we must be careful not to exceed the tolerance range of the selected processes. Table 7 shows the selected processes for dimensions a, c and e and the max and min tolerances obtainable by each, as extracted from Fig. 6 for the corresponding nominal size.

**Table 7. Process Tolerance Limits for the Clutch Assembly**

Part	Dim	Proc	Nom(in)	Sens	Min Tol	Max Tol
Hub	a	Mill	2.1768	-2.6469	0.0025	0.006
Roller	c	Lap	0.9000	-10.548	0.00025	0.00045
Ring	e	Grind	4.0000	2.62721	0.0005	0.0012

### Proportional Scaling by Worst Case:

Since the rollers are vendor-supplied, only tolerances on dimensions a and e may be altered. The proportionality factor P is applied to  $\delta a$  and  $\delta e$ , while  $\delta\phi$  is set to the maximum tolerance of  $\pm 0.017453$  radians ( $\pm 1^\circ$ ).

$$\begin{aligned}
 \delta\phi &= \sum |S_{ij}| \delta x_j & (9) \\
 0.017453 &= |S_{11}| P \delta a + |S_{12}| \delta c + |S_{13}| P \delta e \\
 0.017453 &= 2.6469 \cdot P \cdot 0.004 + 10.5483 \cdot 0.0004 + 2.6272 \cdot P \cdot 0.0008
 \end{aligned}$$

Solving for P:

$$\begin{aligned}
 P &= 1.0429 \\
 \delta a &= 1.0429 \cdot 0.004 = 0.00417 \text{ in} \\
 \delta e &= 1.0429 \cdot 0.0008 = 0.00083 \text{ in}
 \end{aligned}$$

### Proportional Scaling by Root-Sum-Squares:

$$\begin{aligned}
 \delta\phi &= \sqrt{\sum ((S_{ij} \delta x_j)^2)} & (10) \\
 0.017453 &= \sqrt{(S_{11} P \delta a)^2 + (S_{12} \delta c)^2 + (S_{13} P \delta e)^2} \\
 0.017453 &= \sqrt{(-2.6469 \cdot P \cdot 0.004)^2 + (-10.5483 \cdot 0.0004)^2 + (2.6272 \cdot P \cdot 0.0008)^2}
 \end{aligned}$$

Solving for P:

$$\begin{aligned}
 P &= 1.56893 \\
 \delta a &= 1.56893 \cdot 0.004 = 0.00628 \text{ in} \\
 \delta e &= 1.56893 \cdot 0.0008 = 0.00126 \text{ in}
 \end{aligned}$$

Both of these new tolerances exceed the process limits for their respective processes, but by less than 0.001in each. You could round them off to 0.006 and 0.0012. The process limits are not that precise.

### Allocation by Weight Factors:

Grinding the ring is the more costly process of the two. We would like to loosen the tolerance on dimension e. As a first try, let the weight factors be  $w_a = 10$ ,  $w_e = 20$ . This will change the ratio of the two tolerances and scale them to match the 1.0 deg limit. The original tolerances had a ratio of 5:1. The final ratio will be the product of 1:2 and 5:1, or 2.5:1. The sensitivities do not affect the ratio.

$$\begin{aligned} \delta\phi &= \sqrt{\sum((S_{ij} \delta x_j)^2)} & (11) \\ 0.017453 &= \sqrt{(S_{11} P \cdot 10/30 \delta a)^2 + (S_{12} \delta c)^2 + (S_{13} P \cdot 20/30 \delta e)^2} \\ &= \sqrt{(-2.6469 \cdot P \cdot 10/30 \cdot 0.004)^2 + (-10.5483 \cdot 0.0004)^2 + (2.6272 \cdot P \cdot 20/30 \cdot 0.0008)^2} \end{aligned}$$

Solving for P:

$$P = 4.460$$

$$\delta a = 4.460 \cdot 10/30 \cdot 0.004 = 0.00595 \text{ in}$$

$$\delta e = 4.460 \cdot 20/30 \cdot 0.0008 = 0.00238 \text{ in}$$

Evaluating the results, we see that  $\delta a$  is within the 0.006in limit, but  $\delta e$  is well beyond the 0.0012in process limit. But, since  $\delta a$  is so close to its limit, we cannot change the weight factors much without causing  $\delta a$  to go out of bounds. After several trials, the best design seemed to be equal weight factors, which is the same as proportional scaling. We will present a plot later which will make it clear why it turned out this way.

From the preceding examples, we see that the allocation algorithms work the same for 2-D and 3-D assemblies as for 1-D. We simply insert the tolerance sensitivities into the accumulation formulas and carry them through the calculations as constant factors.

### 8.3 Allocation by Cost Minimization

The minimum cost allocation applies equally well to 2-D and 3-D assemblies. If sensitivities are included in the derivation presented in Sec. 6.0, equations 8 through 10 become:

**Table 8. Expressions for Minimum Cost Tolerances in 2-D and 3-D Assemblies**

Worst Case	RSS
$T_i = \left( \frac{k_i B_i S_1}{k_1 B_1 S_i} \right)^{1/(k_i+1)} \cdot T_1^{(k_i+1)/(k_i+1)}$	$T_i = \left( \frac{k_i B_i S_1}{k_1 B_1 S_i} \right)^{1/(k_i+2)} \cdot T_1^{(k_i+2)/(k_i+2)}$
$T_{ASM} = S_1 T_1 + \bullet S_i \left( \frac{k_i B_i S_1}{k_1 B_1 S_i} \right)^{1/(k_i+1)} \cdot T_1^{(k_i+1)/(k_i+1)}$	$T_{ASM}^2 = S_1^2 T_1^2 + \bullet S_i^2 \left( \frac{k_i B_i S_1}{k_1 B_1 S_i} \right)^{2/(k_i+2)} \cdot T_1^{2(k_i+2)/(k_i+2)}$

The cost data for computing process cost is shown in Table 9:

**Table 9. Process Tolerance Cost Data for the Clutch Assembly**

Part	Dim	Proc	Nom(in)	Sens	B	k	Min Tol	Max Tol
Hub	a	Mill	2.1768	-2.6469	0.1018696	-0.45008	0.0025	0.006
Roller	c	Lap	0.9000	-10.548	0.000528	-1.130204	0.00025	0.00045
Ring	e	Grind	4.0000	2.62721	0.0149227	-0.79093	0.0005	0.0012

**Minimum Cost Tolerances by Worst Case:**

To perform tolerance allocation using a Worst Case stackup model, let  $T_1 = \delta a$ , and  $T_i = \delta e$ , then  $S_1 = S_{11}$ ,  $k_1 = k_a$ , and  $B_1 = B_a$ , etc.

$$\begin{aligned}
 T_{ASM} &= |S_{11}| \delta a + |S_{12}| \delta c + |S_{13}| \delta e & (12) \\
 &= |S_{11}| \delta a + |S_{12}| \delta c + |S_{13}| \left( \frac{k_e B_e S_{11}}{k_a B_a S_{13}} \right)^{1/(k_e+1)} \cdot \delta a^{(k_a+1)/(k_e+1)} \\
 0.017453 &= 2.6469 \delta a + 10.5483 \cdot 0.0004 \\
 &+ 2.6272 \left( \frac{0.79093 \cdot 0.0149227 \cdot 2.6469}{0.45008 \cdot 0.1018696 \cdot 2.6272} \right)^{1/(0.20907)} \cdot \delta a^{(0.54992)/(0.20907)}
 \end{aligned}$$

The only unknown is  $\delta a$ , which may be found by iteration.  $\delta e$  may then be found once  $\delta a$  is known. Solving for  $\delta a$  and  $\delta e$ :

$$\begin{aligned}
 \delta a &= 0.00198 \text{ in} & (13) \\
 \delta e &= \left( \frac{0.79093 \cdot 0.0149227 \cdot 2.6469}{0.45008 \cdot 0.1018696 \cdot 2.6272} \right)^{1/(0.20907)} \cdot 0.00198^{(0.54992)/(0.20907)} \\
 &= 0.00304 \text{ in}
 \end{aligned}$$

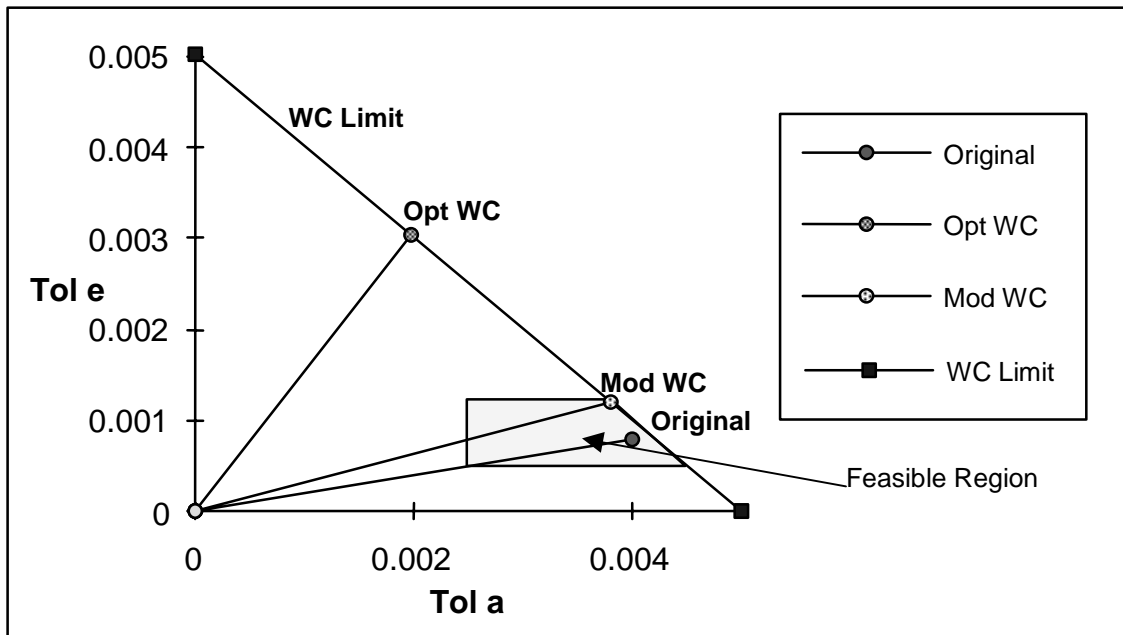
The cost corresponding to holding these tolerances would be reduced from  $C = \$5.42$  to  $C = \$3.14$ .

Comparing these values to the process limits in Table 9, we see that  $\delta a$  is below its lower process limit ( $0.0025 < \delta a < 0.006$ ), while  $\delta e$  is much larger than the upper process limit ( $0.0005 < \delta e < 0.0012$ ). If we decrease  $\delta e$  to the upper process limit,  $\delta a$  can be increased until  $T_{ASM}$  equals the spec limit. The resulting values and cost are then:

$$\delta a = 0.0038 \text{ in} \quad \delta e = 0.0012 \text{ in} \quad C = \$4.30$$

The relationship between the resulting three pairs of tolerances is very clear when they are plotted as shown in Fig. 14. Tol e and Tol a are plotted as points in 2-D tolerance space. The feasible region is bounded by a box formed by the upper and lower process limits, which is cut off by the Worst Case limit curve. The original tolerances of (0.004, 0.0008) lie within the feasible region, nearly touching the WC Limit. Extending a line through the original tolerances to the WC Limit yields the proportional scaling results found in Section 2.0 (0.00417, 0.00083), which is not much improvement over the original tolerances. The minimum cost tolerances (OptWC) were a significant change, but moved outside the feasible region. The feasible point of lowest cost (Mod WC) resulted at the intersection of the upper limit for Tol e and the WC Limit (0.0038, 0.0012).

This type of plot really clarifies the relationship between the three results. Unfortunately, it is limited to a 2-D graph, so it is only applicable to an assembly with two design tolerances.



**Figure 14.** Tolerance allocation results for a Worst Case model.

**Minimum Cost Tolerances by RSS:**

Repeating the minimum cost tolerance allocation using the RSS stackup model:

$$\begin{aligned}
 T_{ASM}^2 &= (S_{11} \delta a)^2 + (S_{12} \delta c)^2 + (S_{13} \delta e)^2 & (14) \\
 &= (S_{11} \delta a)^2 + (S_{12} \delta c)^2 + S_{13}^2 \left( \frac{k_e B_e S_{11}}{k_a B_a S_{13}} \right)^{2/(k_e+2)} \cdot \delta a^{2(k_a+2)/(k_e+2)} \\
 (0.017453)^2 &= (2.6469 \delta a)^2 + (10.5483 \cdot 0.0004)^2 \\
 &+ 2.6272^2 \left( \frac{0.79093 \cdot 0.0149227 \cdot 2.6469}{0.45008 \cdot 0.1018696 \cdot 2.6272} \right)^{2/(1.20907)} \cdot \delta a^{2(1.54992)/(1.20907)}
 \end{aligned}$$

Solving for  $\delta a$  by iteration and  $\delta e$  as before:

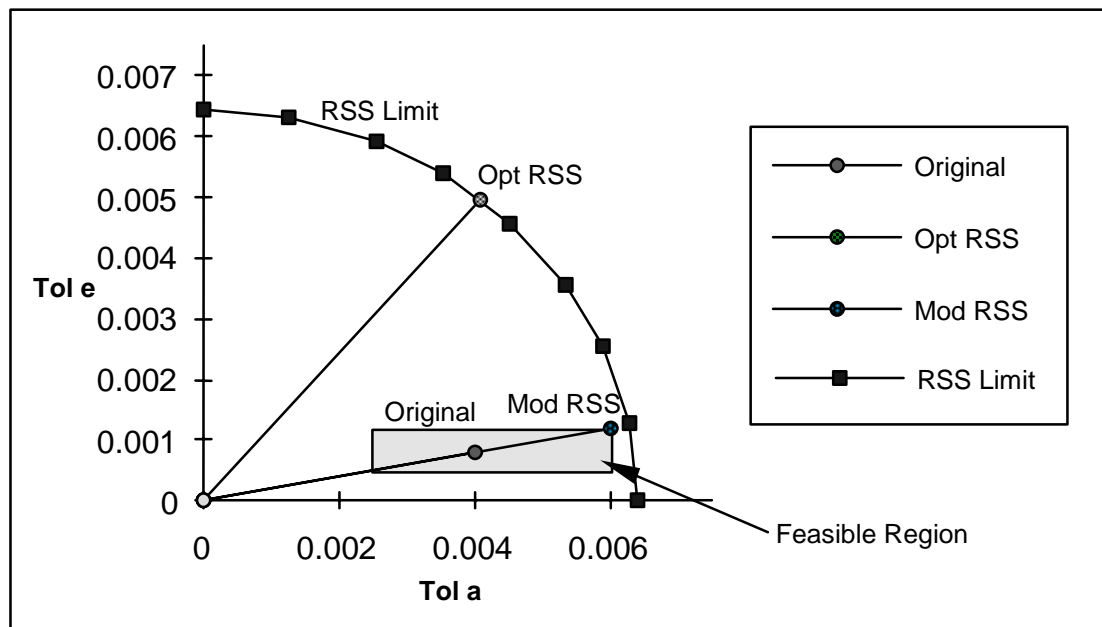
$$\begin{aligned}
 \delta a &= 0.00409 \text{ in} & (15) \\
 \delta e &= \left( \frac{0.79093 \cdot 0.0149227 \cdot 2.6469}{0.45008 \cdot 0.1018696 \cdot 2.6272} \right)^{1/(1.20907)} \cdot 0.00409^{(0.54992)/(1.20907)} \\
 &= 0.00495 \text{ in}
 \end{aligned}$$

The cost corresponding to holding these tolerances would be reduced from C= \$5.42 to C= \$2.20.

Comparing these values to the process limits in Table 9, we see that  $\delta a$  is now safely within its process limits ( $0.0025 < \delta a < 0.006$ ), while  $\delta e$  is still much larger than the upper process limit ( $0.0005 < \delta e < 0.0012$ ). If we again decrease  $\delta e$  to the upper process limit as before,  $\delta a$  can be increased until it equals the upper process limit. The resulting values and cost are then:

$$\delta a = 0.006 \text{ in} \quad \delta e = 0.0012 \text{ in} \quad C = \$4.07$$

The plot in Fig. 15 shows the three pairs of tolerances. The box containing the feasible region is entirely within the RSS Limit curve. The original tolerances of (0.004, 0.0008) lie near the center of the feasible region. Extending a line through the original tolerances to the RSS Limit yields the proportional scaling results found in Sec. 2.0 (0.00628, 0.00126), both of which lie just outside the feasible region. The minimum cost tolerances (OptRSS) were a significant change, but moved far outside the feasible region. The feasible point of lowest cost (ModRSS) resulted at the upper limit corner of the feasible region (0.006, 0.0012).



**Figure 15.** Tolerance allocation results for the RSS model.

Comparing Figs. 14 and 15, we see that the RSS Limit curve intersects the horizontal and vertical axes at values greater than 0.006 in, while the WC Limit curve intersects near 0.005 in. tolerance. The intersections are found by letting Tol a or Tol e go to zero in the equation for  $T_{ASM}$  and solving for the remaining tolerance. The RSS and WC Limit curves do not converge to the same point because the fixed tolerance  $\delta c$  is subtracted from  $T_{ASM}$  differently for WC than RSS.

### 9.0 Tolerance Allocation with Process Selection

Examining Fig. 15 further, the feasible region appears very small. There is not much room for tolerance design. The optimization preferred to drive Tol e to a much larger value. One way to enlarge the feasible region is to select an alternate process for dimension e. Instead of grinding, suppose we consider turning. The process limits change to  $(0.002 < \delta e < 0.008)$ , with  $B_e = 0.118048$   $k_e = -0.45747$ . Table 10 shows the revised data.

**Table 10. Revised Process Tolerance Cost Data for the Clutch Assembly**

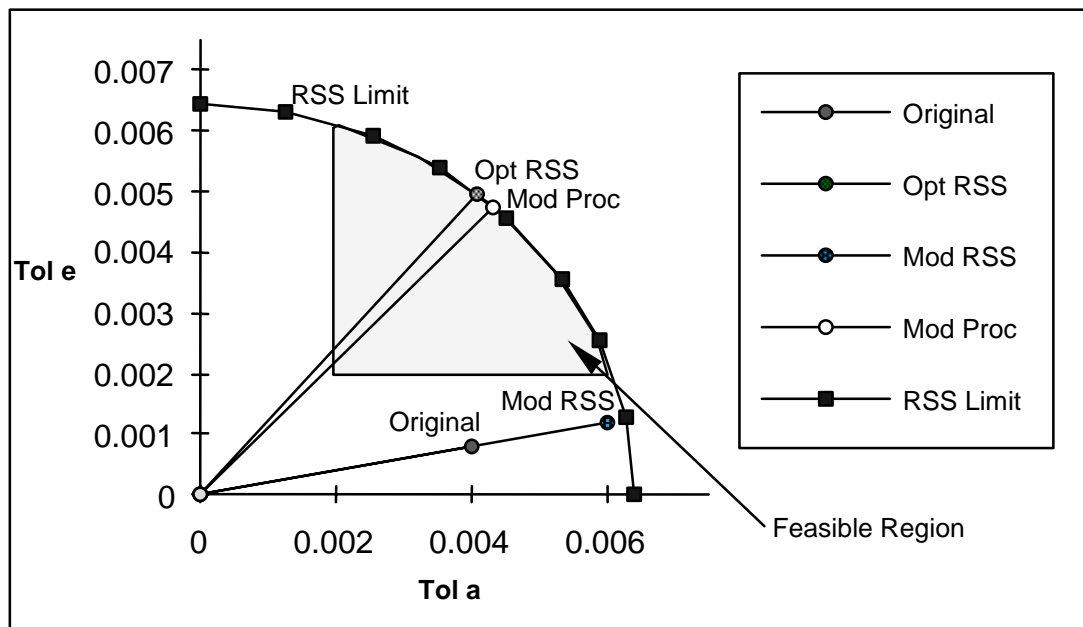
Part	Dim	Proc	Nom(in)	Sens	B	k	Min Tol	Max Tol
Hub	a	Mill	2.1768	-2.6469	0.1018696	-0.45008	0.0025	0.006
Roller	c	Lap	0.9000	-10.548	0.000528	-1.130204	0.00025	0.00045
Ring	e	Turn	4.0000	2.62721	0.118048	-0.45747	0.002	0.008

Milling and turning are processes with nearly the same precision. Thus,  $B_e$  and  $B_a$  are nearly equal as are  $k_e$  and  $k_a$ . The resulting RSS allocated tolerances and cost are:

$$\delta a = 0.00434 \text{ in} \quad \delta e = 0.00474 \text{ in} \quad C = \$2.54$$

The new optimization results are shown in Fig. 16. The feasible region is clearly much larger and the minimum cost point (Mod Proc) is on the RSS Limit curve on the region boundary. The new optimum point has also changed from the previous result (Opt RSS) due the change in  $B_e$  and  $k_e$  for the new process.



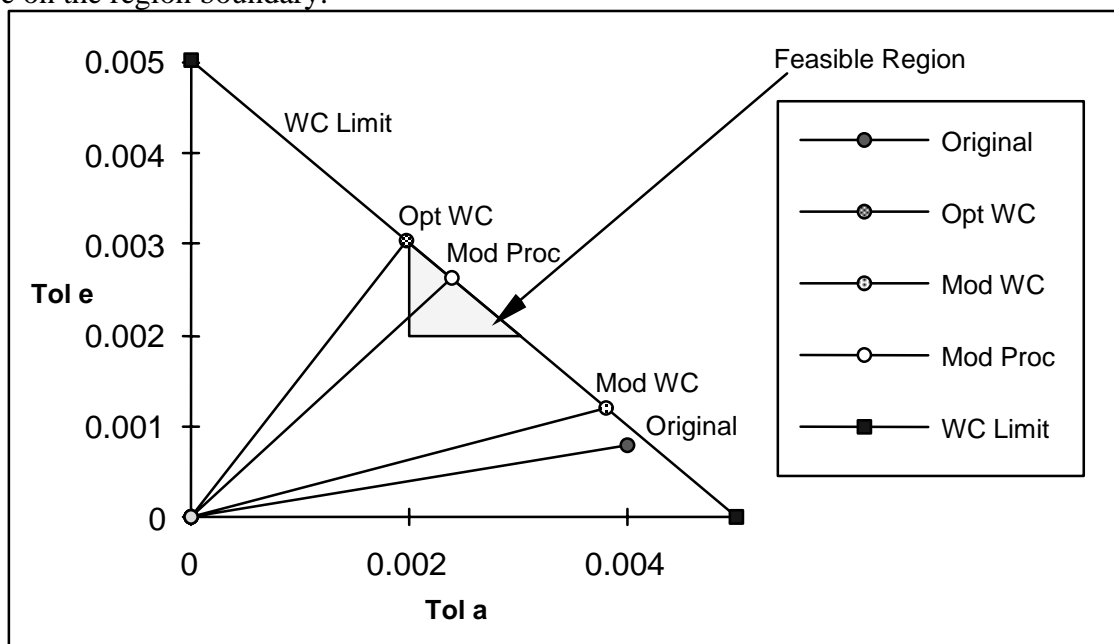


**Figure 16.** Tolerance allocation results for the modified RSS model.

The resulting WC allocated tolerances and cost are:

$$\delta a = 0.00240 \text{ in} \quad \delta e = 0.00262 \text{ in} \quad C = \$3.33$$

The modified optimization results are shown in Fig. 17. The feasible region is the smallest yet due to the tight Worst Case Limit. The minimum cost point (Mod Proc) is on the WC Limit curve on the region boundary.



**Figure 17.** Tolerance allocation results for the modified WC model.

Including cost functions with tolerance selection makes possible the quantitative comparison of alternate processes to see if cost reductions could be achieved by a change in process. If cost vs. tolerance data are available for a full range of processes, process selection can even be automated. A very systematic and efficient search technique, which automates this task has been published [Chase, et al., 1989]. It compares several methods for including process selection in tolerance allocation and gives a detailed description of the one found to be most efficient.

## 10.0 Summary

The results of WC and RSS cost allocation of tolerances are summarized in the two bar charts, Figs. 18 and 19. The changes in magnitude of the tolerances is readily apparent. Costs have been added for comparison.

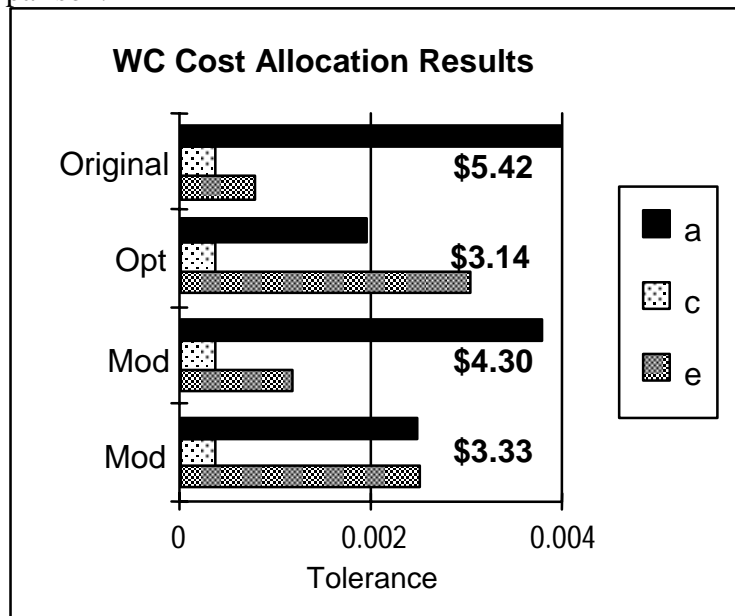


Figure 18. Tolerance allocation results for the WC model.

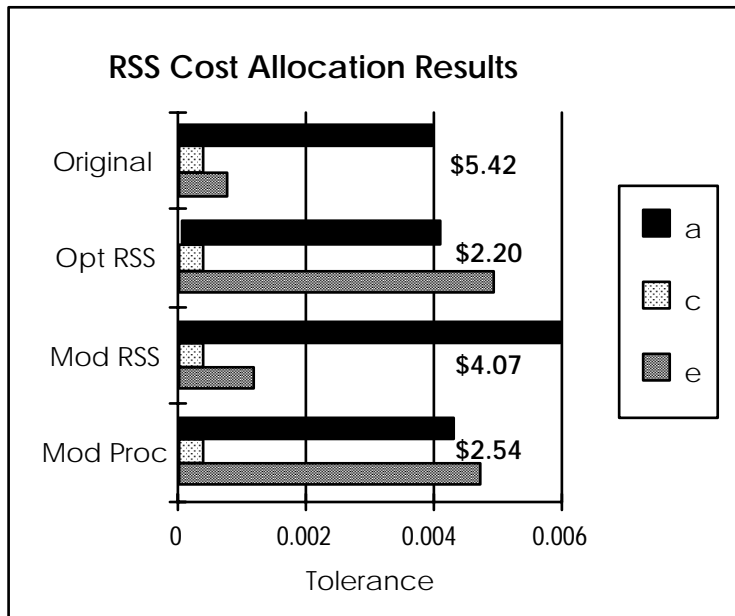


Figure 19 Tolerance allocation results for the RSS model.

Summarizing, the original tolerances for both WC and RSS were safely within tolerance constraints, but the costs were high. Optimization reduced the cost dramatically, however, the resulting tolerances exceeded the recommended process limits. The modified WC and RSS tolerances were adjusted to conform to the process limits, resulting in a moderate decrease in cost, about 20%. Finally, the effect of changing processes was illustrated, which resulted in a cost reduction near the first optimization, only the allocated tolerances remained in the new feasible region.

A designer would probably not attempt all of these cases in a real design problem. He would be wise to rely on the RSS solution, possibly trying WC analysis for a case or two for comparison. Note that the clutch assembly only had three dimensions contributing to the tolerance stack. If there had been six or eight, the difference between WC and RSS would have been much more significant.

It should be noted that tolerances specified at the process limit may not be desirable. If the process is not well controlled, it may be difficult to hold it at the limit. In such cases, the designer may want to back off from the limits to allow for process uncertainties.

## 11.0 References

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## APPENDIX

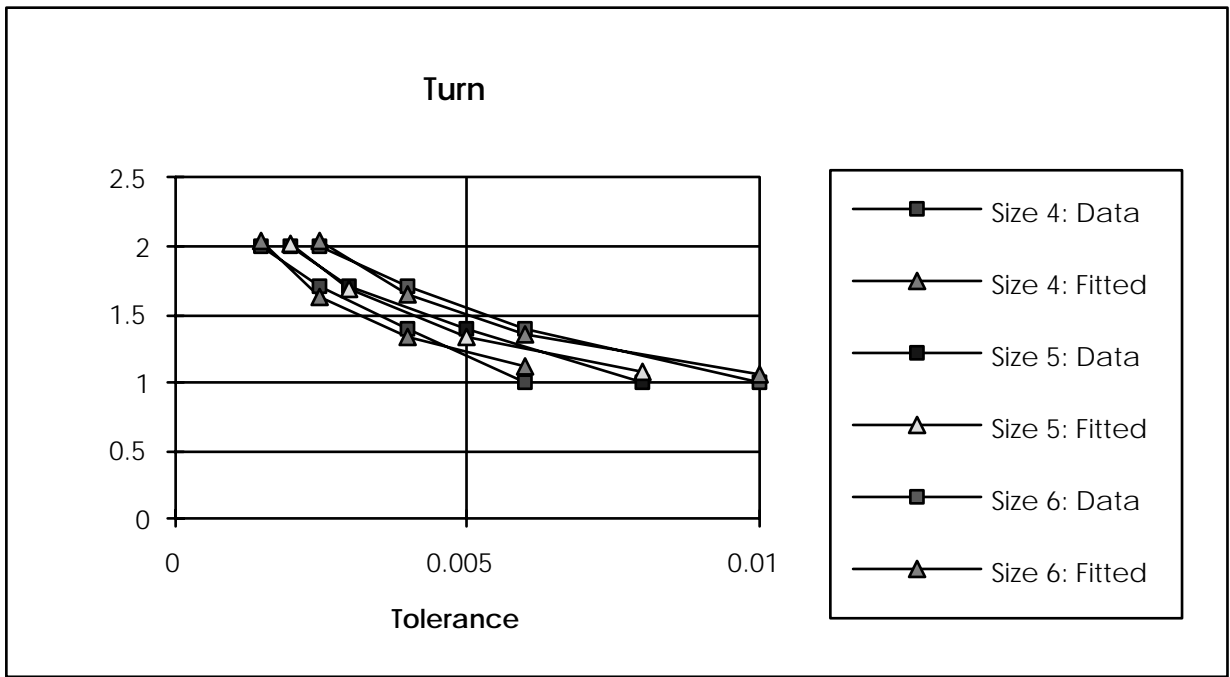
### Cost-Tolerance Functions for Metal Removal Processes

Although it is well known that tightening tolerances increases cost, adjusting the tolerances on several components in an assembly and observing its effect on cost is an impossible task. Until you have a mathematical model, you can not effectively optimize the allocation of tolerance in an assembly. Elegant tools for minimum cost tolerance allocation have been developed over several decades. However, they require empirical functions describing the relationship between tolerance and cost.

Cost vs. tolerance data is very scarce. Very few companies or agencies have attempted to gather such data. Companies who do consider it proprietary, so it is not published. The data is site and machine-specific and subject to obsolescence due to inflation. In addition, not all processes are capable of continuously adjustable precision.

Metal removal processes have the capability to tighten or loosen tolerances by changing feeds, speeds, and depth of cut or by modifying tooling, fixtures, cutting tools and coolants. The workpiece may also be modified by switching to a more machinable alloy or modifying geometry to achieve greater rigidity.

A noteworthy study by the U.S. Army in the 1940s experimentally determined the natural tolerance range for the most common metal removal processes [4]. They also compared the cost of the various processes and the relative cost of tightening tolerances. Relative costs were used to eliminate the effects of inflation. The resulting chart, Table. A-1, appears in references [1 and 2]. Least squares curve fits were performed at BYU and are presented here for the first time. The Reciprocal Power equation,  $C = A + B/T^k$ , presented in Chapter 12, was used as the empirical function. Fig. A-1 shows a typical plot of the original data and the fitted data. The curve fit procedure was a standard nonlinear method described in reference [3], which uses weighted logarithms of the data to convert to a linear regression problem. Results are tabulated in Table A-2 and plotted in Figs. A-2 and A-3.



**Fig. A-1** Plot of cost vs tolerance for fitted and raw data for the turning process

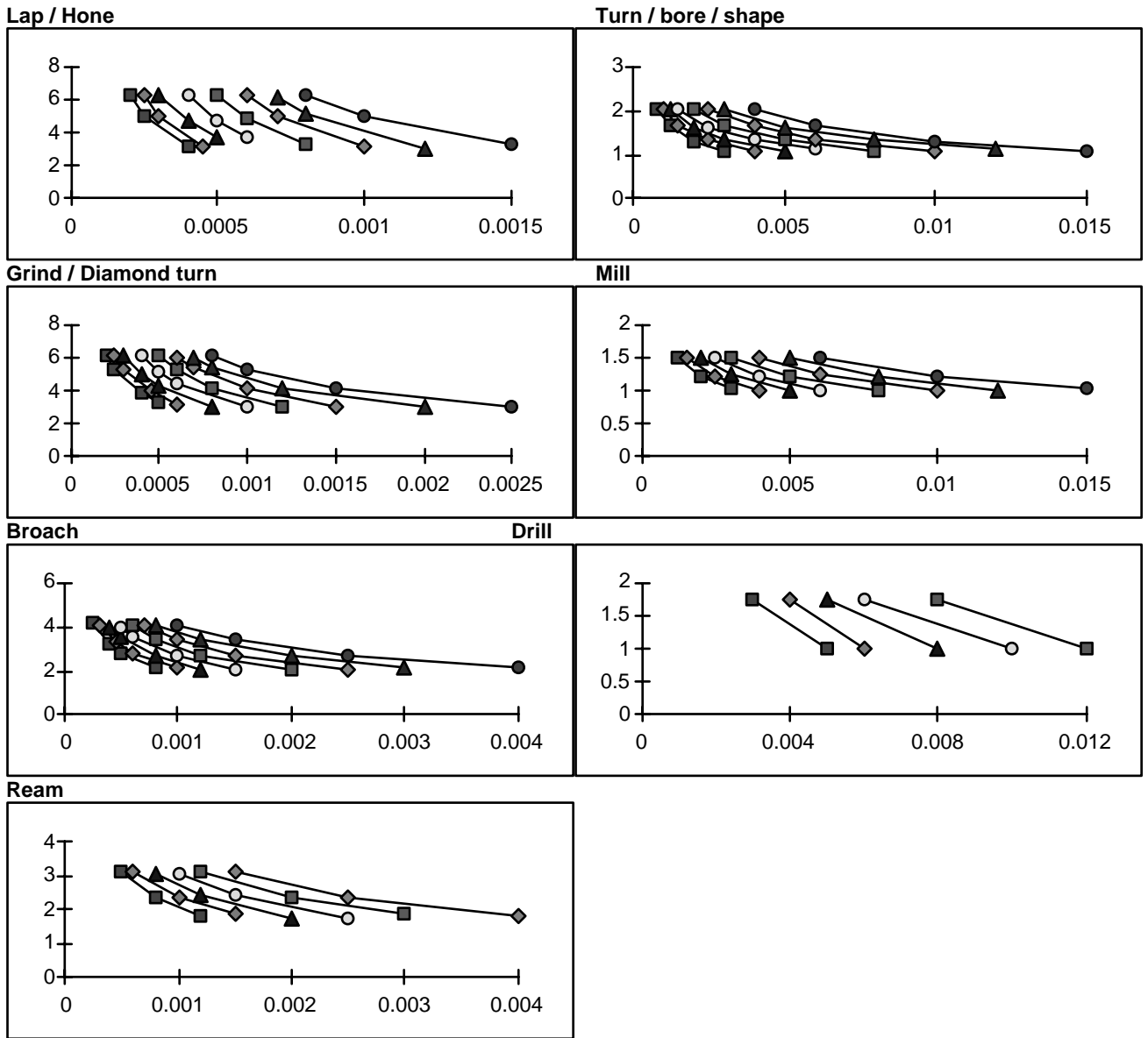
**Table A-1 Relative Cost of Obtaining Various Tolerance Levels**

Range of Sizes (in.)											
From	To	Tolerances (in.)									
0.000	0.599	0.0002	0.00025	0.0004	0.0005	0.0008	0.0012	0.0020	0.0030	0.0050	
0.600	0.999	0.00025	0.0003	0.00045	0.0006	0.0010	0.0015	0.0025	0.0040	0.0060	
1.000	1.499	0.0003	0.0004	0.0005	0.0008	0.0012	0.0020	0.0030	0.0050	0.0080	
1.500	2.799	0.0004	0.0005	0.0006	0.0010	0.0015	0.0025	0.0040	0.0060	0.0100	
2.800	4.499	0.0005	0.0006	0.0008	0.0012	0.0020	0.0030	0.0050	0.0080		
4.500	7.799	0.0006	0.0007	0.0010	0.0015	0.0025	0.0040	0.0060	0.0100		
7.800	13.599	0.0007	0.0008	0.0012	0.0020	0.0030	0.0050	0.0080	0.0120		
13.600	20.999	0.0008	0.0010	0.0015	0.0025	0.0040	0.0060	0.0100	0.0150		
21.00 and over follow same tolerancing trends											
Process		Relative Cost of Tightening Tolerance*									Process Cost
Lap and Hone		200%	180%	100%							300%
Grind, Diamond Turn and Bore		200%	180%	140%	100%						300%
Broach			200%	175%	140%	100%					200%
Ream					175%	140%	100%				175%
Turn, Bore, Slot, Plane, and Shape						200%	170%	140%	100%		100%
Mill							150%	125%	100%		100%
Drill									175%	100%	100%

\*Total relative cost for a given process is the percentage product of the tolerance tightening cost and the process cost (200%\*300%=600%).  
 Reproduced from reference [2]

**Table A-2 Cost-Tolerance Functions for Metal Removal Processes**

Size Range	A	B	k	Min Tol	Max Tol
<b>Lap / Hone</b>					
0.000-0.599		0.00189378	0.9508781	0.0002	0.0004
0.600-0.999		0.00052816	1.1302036	0.00025	0.00045
1.000-1.499		0.00220173	0.9808618	0.0003	0.0005
1.500-2.799		0.00033129	1.2590875	0.0004	0.0006
2.800-4.499		0.00026156	1.3269297	0.0005	0.0008
4.500-7.799		0.00038119	1.3073528	0.0006	0.001
7.800-13.599		0.00059824	1.2716314	0.0007	0.0012
13.600-20.999		0.00427422	1.0221757	0.0008	0.0015
<b>Grind / Diamond turn</b>					
0.000-0.599		0.02484363	0.6465727	0.0002	0.0005
0.600-0.999		0.01525616	0.7221989	0.00025	0.0006
1.000-1.499		0.0205072	0.7039047	0.0003	0.0008
1.500-2.799		0.0133561	0.7827624	0.0004	0.001
2.800-4.499		0.01492268	0.790932	0.0005	0.0012
4.500-7.799		0.02467047	0.7413291	0.0006	0.0015
7.800-13.599		0.05119944	0.6548091	0.0007	0.002
13.600-20.999		0.08317908	0.6017646	0.0008	0.0025
<b>Broach</b>					
0.000-0.599		0.0438552	0.548619	0.00025	0.0008
0.600-0.999		0.04670538	0.55230115	0.0003	0.001
1.000-1.499		0.04071362	0.58686634	0.0004	0.0012
1.500-2.799		0.048524	0.579761	0.0005	0.0015
2.800-4.499		0.0637591	0.559608	0.0006	0.002
4.500-7.799		0.0922923	0.521758	0.0007	0.0025
7.800-13.599		0.144046	0.46957	0.0008	0.003
13.600-20.999		0.171785	0.45907	0.001	0.004
<b>Ream</b>					
0.000-0.599		0.03245261	0.6000163	0.0005	0.0012
0.600-0.999		0.04682158	0.565492	0.0006	0.0015
1.000-1.499		0.04204992	0.6021191	0.0008	0.002
1.500-2.799		0.04809684	0.6021191	0.001	0.0025
2.800-4.499		0.06929088	0.565492	0.0012	0.003
4.500-7.799		0.09203907	0.5409254	0.0015	0.004
<b>Turn / bore / shape</b>					
0.000-0.599		0.07201641	0.46822793	0.0008	0.003
0.600-0.999		0.085969502	0.45747142	0.001	0.004
1.000-1.499		0.101233386	0.44723008	0.0012	0.005
1.500-2.799		0.11800302	0.4389869	0.0015	0.006
2.800-4.499		0.11804756	0.45747142	0.002	0.008
4.500-7.799		0.12576137	0.46536684	0.0025	0.01
7.800-13.599		0.15997103	0.4389869	0.003	0.012
13.600-20.999		0.15300611	0.46822793	0.004	0.015
<b>Mill</b>					
0.000-0.599		0.0862308	0.4259173	0.0012	0.003
0.600-0.999		0.10878812	0.4044547	0.0015	0.004
1.000-1.499		0.09544417	0.4431399	0.002	0.005
1.500-2.799		0.10186958	0.4500798	0.0025	0.006
2.800-4.499		0.14399071	0.4044547	0.003	0.008
4.500-7.799		0.12976209	0.4431399	0.004	0.01
7.800-13.599		0.13916564	0.4500798	0.005	0.012
13.600-20.999		0.17114563	0.4259173	0.006	0.015
<b>Drill</b>					
0.000-0.599		0.00301435	1.0955124	0.003	0.005
0.600-0.999		0.00085791	1.3801824	0.004	0.006
1.000-1.499		0.00318631	1.1906627	0.005	0.008
1.500-2.799		0.00644133	1.0955124	0.006	0.01
2.800-4.499		0.00223316	1.3801824	0.008	0.012



**Figure A-2 Plot of Fitted Cost vs. Tolerance Functions**



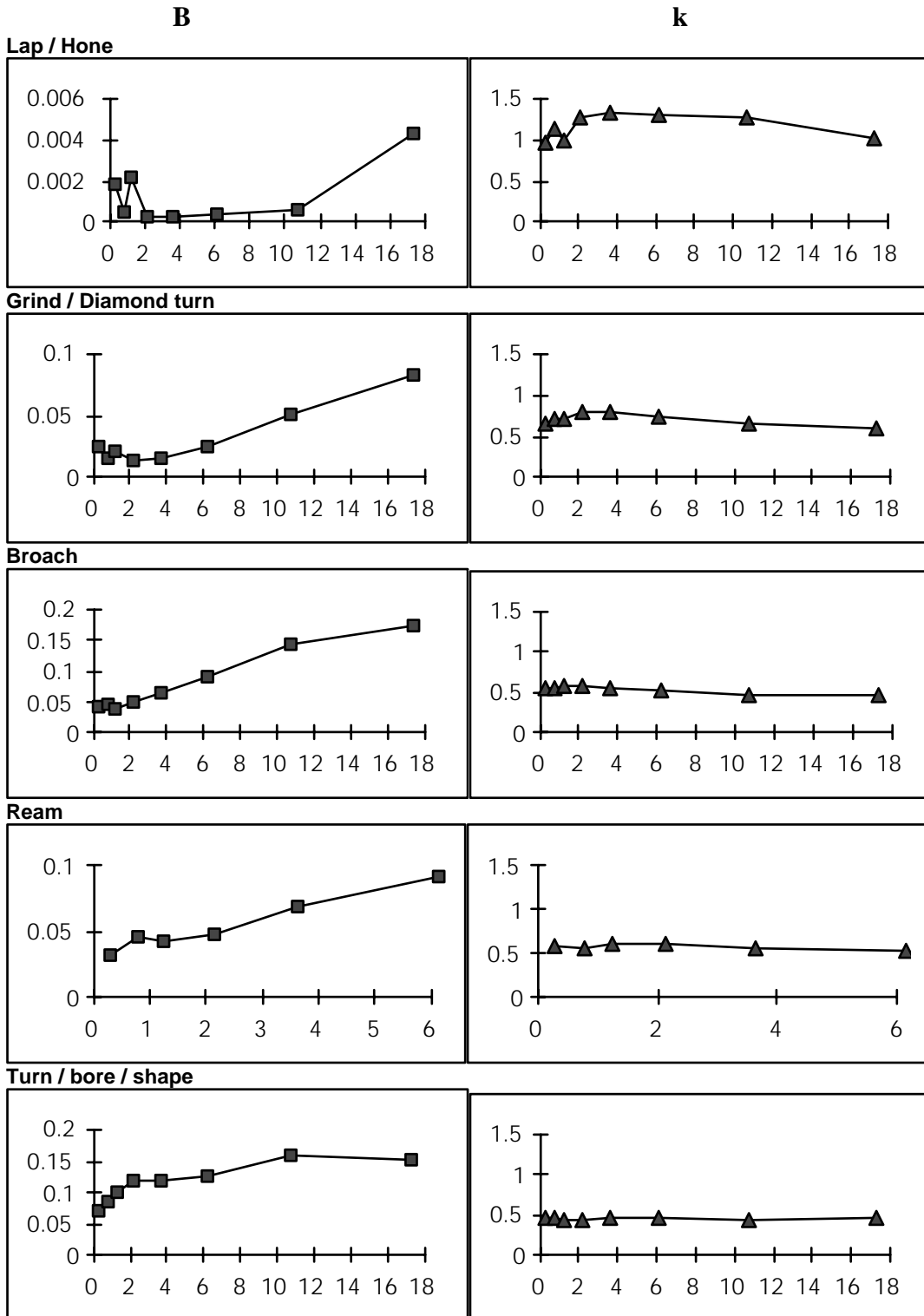
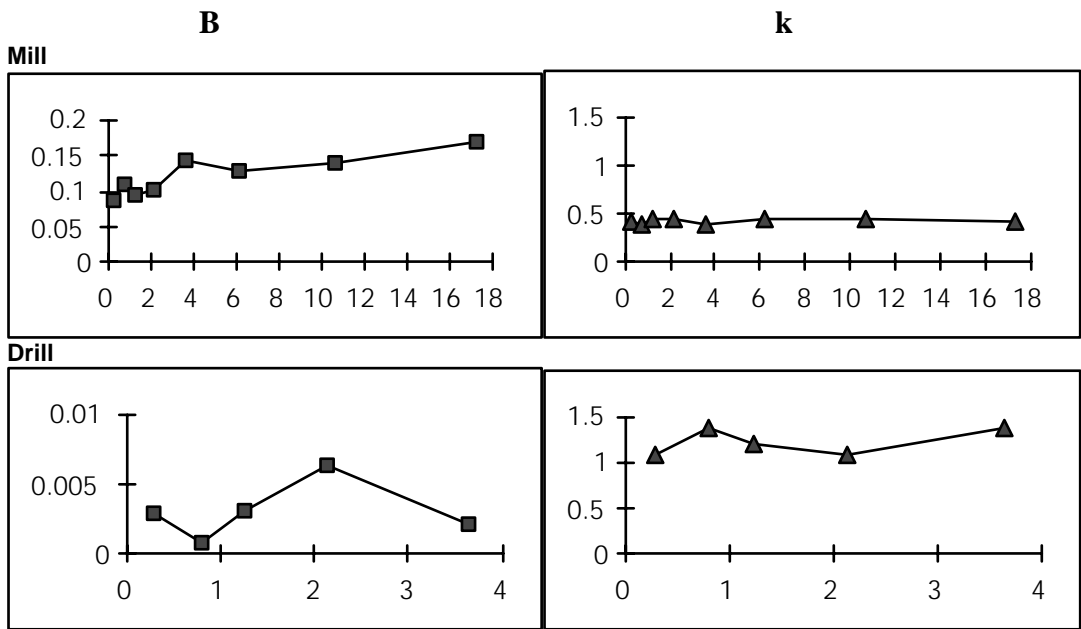


Figure A-3 Plot of Coefficients vs. Size for Cost-Tolerance Functions



**Figure A-3 (continued) Plot of Coefficients vs. Size for Cost-Tolerance Functions**

Curve fits were performed by mechanical engineering student, David Todd.

**References:**

1. Hansen, Bertrand L., Quality Control: Theory and Applications, Prentice-Hall, Upper Saddle River, NJ, 1963.
2. Jamieson, Archibald, Introduction to Quality Control, Reston Publishing, Reston, VA, 1982.
3. Pennington, Ralph H., Introductory Computer Methods and Numerical Analysis, 2nd ed, MacMillan, Toronto, Canada, 1970.
4. U.S.Army Management Engineering Training Activity, Rock Island Arsenal, IL. (Original report is out of print)